1. Write a Matlab function \([t,y] = \text{bvpolve}(u,v,w,a,b,alpha,\beta,m)\) to solve a linear two-point boundary value problem of the form

\[
y''(t) = u(t) + v(t)y(t) + w(t)y'(t) \\
y(a) = \alpha, \quad y(b) = \beta
\]

with the finite difference method. Use the subroutine \texttt{tridiag.m}\ presented in class to solve the tridiagonal linear system.

Assume that the functions \(u, v\) and \(w\) are defined separately, e.g., in a driver script.

2. Consider the problem

\[
t^2y''(t) - t(t + 2)y'(t) + (t + 2)y(t) = 0
\]

whose general solution is given by \(y(t) = c_1 t + c_2 te^t\).

(a) What is the solution if the boundary conditions

\[y(1) = e, \quad y(2) = 2e^2\]

are used?

(b) Test your code from Exercise 1 with this problem. Plot the approximate and exact solutions together for \(m = 19\).

(c) Perform a series of experiments with \(m = 4, 9, 19, 39, 79\), compute the maximum error and observe how it changes with \(m\) (or \(h\)).

3. Repeat Exercise 2 for the problem

\[
y''(t) + 2y'(t) + 10t = 0 \\
y(0) = 1, \quad y(1) = 2
\]

whose general solution is given by \(y(t) = -\frac{5}{2}t^2 + \frac{5}{2}t + c_1 e^{-2t} + c_2\).