Friday, September 26, 2014

If n is a positive integer such that $2n + 1$ is a perfect square, show that $n + 1$ is the sum of two successive perfect squares.

Solution. Assume that $2n + 1 = s^2$ where s and n are positive integers. Since s^2 is odd, we must have that s is also odd (since the square of an even integer is even). Thus $s = 2t + 1$ for some integer t. Hence

$$2n + 1 = (2t + 1)^2 = 4t^2 + 4t + 1$$

which yields that

$$n = 2t^2 + 2t ,$$

and hence that

$$n + 1 = 2t^2 + 2t + 1 = t^2 + (t + 1)^2 .$$

Good Luck! Have fun and enjoy Mathematics!