Friday, October 31, 2014

Let

$$f(t) = \int_1^t \int_2^x \int_3^y g'(z) dz dy dx,$$

where g is a function with a continuous derivative. Express $f''(t)$ as simply as you can in terms of g.

Solution. We'll show that $f''(t) = g(t) - g(3)$. By the Fundamental Theorem of Calculus we have $\int_3^y g'(z)dz = g(y) - g(3)$, so

$$f(t) = \int_1^t \int_2^x [g(y) - g(3)] dy dx = \int_1^t h(x) dx,$$

where $h(x) = \int_2^x [g(y) - g(3)]dy$. By the FTC again,

$$f'(t) = h(t) = \int_2^t [g(y) - g(3)] dy,$$

and

$$f''(t) = g(t) - g(3).$$

Good Luck! Have fun and enjoy Mathematics!