Friday, October 19, 2012
The decimals of a given natural number $n$ contain the digits 1,3,7 and 9. Prove that it is possible to rearrange the digits of $n$ such that the new number is divisible by 7.

Solution. Without loss of generality we can consider that $n$ has the last digits equal to 1,3,7,9, else rearrange first. Hence, $n$ can be written as a sum of 1379 and $a$ where $a$ has last four digits equal to zero. Note that, the following numbers

$$1379, \ 1793, \ 3719, \ 1739, \ 1397, \ 1937, \ 1973,$$

have the reminders of division by 7 equal to 0,1,2,3,4,5,6. Thus, if $a$ has a reminder after division by 7 equal to $r$, then take that four digit number from above, than gives the reminder $7 - r$. Now, $a$ plus the chosen four digit number will be divisible by 7.

Good Luck! Have fun and enjoy Mathematics!