Friday, October 17, 2014

Can you find an integer number such that after deleting the first digit the obtained number is 57 times smaller? Can it be 58 times smaller?

Solution. There exists such number, for example 7125 satisfies $7125/125 = 57$. However there is no number that is 58 times smaller. Indeed, assume that such number exists, and let us denote by x the deleted digit, by k the number of remaining digits, and by y the number obtained after deletion. Thus,

$$x \cdot 10^k + y = 58y,$$

that implies that

$$x \cdot 10^k = 57y.$$

The right hand side is divisible by 19, while the left hand side is not divisible by 19. Hence, contradiction.

Good Luck! Have fun and enjoy Mathematics!