Friday, October 10, 2014

Let $f(x) = \frac{1}{1-x^2}$. Find a formula for $f^{(n)}(x)$, the n^{th} derivative of f at x.

Solution. By partial fraction decomposition, we find that

$$f(x) = \frac{1}{2}\left(\frac{1}{1-x} + \frac{1}{1+x}\right) .$$

Hence

$$f^{(n)}(x) = \frac{1}{2}\left(\frac{d^n}{dx^n}(1-x)^{-1} + \frac{d^n}{dx^n}(1+x)^{-1}\right) .$$

It is easy to verify by induction that

$$\frac{d^n}{dx^n}(1-x)^{-1} = \frac{n!}{(1-x)^{n+1}} \qquad \text{and} \qquad \frac{d^n}{dx^n}(1+x)^{-1} = \frac{(-1)^n n!}{(1+x)^{n+1}} .$$

Thus,

$$f^{(n)}(x) = \frac{n!}{2}\left(\frac{1}{(1-x)^{n+1}} + \frac{(-1)^n}{(1+x)^{n+1}}\right) .$$

Good Luck! Have fun and enjoy Mathematics!