

Friday, November 07, 2014

Is it possible to draw in space 15 radii from a point such that the angle between any two radii is more than 60 degrees?

Solution. It is not possible. Consider a sphere of radius R centered at that point. For each radius, consider a cone with vertex at the center of the sphere, and such that the angle at the vertex is equal to 60 degrees and the radius is the axis of the cone. This cone will cut a part of the sphere that has surface area equal to $2\pi Rh$, where $h = R(1 - \sqrt{3}/2)$. On the other hand, the surface area of the sphere is equal to $4\pi R^2$, which is smaller than the 15 parts cut by the cones:

$$15 \cdot 2\pi R^2 \left(1 - \frac{\sqrt{3}}{2}\right) > 4\pi R^2$$

or

$$15\left(1 - \frac{\sqrt{3}}{2}\right) > 2,$$

which is easy to check that holds true.

Good Luck! Have fun and enjoy Mathematics!