ILLINOIS INSTITUTE OF TECHNOLOGY

 Department of Applied Mathematics
 and
 IIT SIAM Student Chapter

Math Weekly Problem Competition

Friday, March 20, 2015

If a, b, c are positive numbers such that (1 + a)(1 + b)(1 + c) = 8, prove $abc \le 1$.

Solution.

Assume that (1 + a)(1 + b)(1 + c) = 8, and hence that

1 + a + b + c + ab + ac + bc + abc = 8.

By the arithmetic-mean-geometric-mean inequality

$$1 = \frac{1 + a + b + c + ab + ac + bc + abc}{8} \ge \sqrt[8]{1 \cdot a \cdot b \cdot c \cdot ab \cdot ac \cdot bc \cdot abc}$$
$$= \sqrt[8]{a^4 b^4 c^4} = \sqrt{abc}$$

from which $abc \leq 1$ follows easily.

Good Luck! Have fun and enjoy Mathematics!

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