ILLINOIS INSTITUTE OF TECHNOLOGY Department of Applied Mathematics and IIT SIAM Student Chapter

Math Weekly Problem Competition

Friday, March 06, 2015

Sum the infinite series
$$\sum_{i=1}^{\infty} \frac{1}{(3i-2)(3i+1)}$$

Solution. By the method of partial fraction decomposition we find that

$$\frac{1}{(3i-2)(3i+1)} = \frac{1/3}{3i-2} - \frac{1/3}{3i+1} \; .$$

Hence

$$\sum_{i=1}^{n} \frac{1}{(3i-2)(3i+1)} = \frac{1}{3} \sum_{i=1}^{n} \left(\frac{1}{3i-2} - \frac{1}{3i+1} \right)$$
$$= \frac{1}{3} \left(\frac{1}{1} - \frac{1}{3n+1} \right)$$

because the summation telescopes. Thus,

$$\sum_{i=1}^{\infty} \frac{1}{(3i-2)(3i+1)} = \lim_{n \to \infty} \frac{1}{3} \left(\frac{1}{1} - \frac{1}{3n+1} \right) = \frac{1}{3}$$

Good Luck! Have fun and enjoy Mathematics!

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