## Friday, February 21, 2014

A car travels from point $A$ ($x = 0$) to the point $B$ ($x = L$). We know the car velocity $v$ as the function of the distance $x$ for any $0 \le x \le L$. Find the formula for computing the total time the car takes traveling from $A$ to $B$. Show the details of your derivation. Bonus point: do you need any additional assumption for your formula to be valid?

**Solution.** Denote the total time by $T$.

$$T = \int_0^T \mathrm{d}t = \int_0^L \frac{\mathrm{d}t}{\mathrm{d}x}\mathrm{d}x = \int_0^L \frac{1}{\frac{\mathrm{d}x}{\mathrm{d}t}}\mathrm{d}x = \int_0^L \frac{1}{v(x)}\mathrm{d}x,$$

where we have used the derivative formula for an inverse function $\dfrac{\mathrm{d}t}{\mathrm{d}x} = \dfrac{1}{\frac{\mathrm{d}x}{\mathrm{d}t}}$. The assumption of the derivation is that we have to assume the existence of the inverse function $x = x(t)$, in the other words, the car does not stop at any point.

Good Luck! Have fun and enjoy Mathematics!