Friday, February 13, 2015

Find all integer solutions to the equation $x^2 + y^2 = 1000003$.

Solution. Consider the problem modulo 4: x^2 can only be 0 (if x is even) or 1 (if x is odd, since $(2k+1)^2 = 4k^2 + 4k + 1$), and likewise for y^2. So the left side is 0,1, or 2, but the right side is 3. Hence, there are no solutions.

Good Luck! Have fun and enjoy Mathematics!