

**Weekly Problem Competition**

**Friday, October 16, 2009**

Assume that  $a_1, a_2, \dots, a_n$  are positive integers ( $n \geq 2$ ), such that  $a_1 < a_2 < \dots < a_n$  and  $\sum_{k=1}^n \frac{1}{a_k} \leq 1$ . Prove that for any real number  $x$ , the following inequality holds true

$$\left( \sum_{k=1}^n \frac{1}{a_k^2 + x^2} \right)^2 \leq \frac{1}{2} \frac{1}{a_1(a_1 - 1) + x^2}.$$

**Remarks:**

The rules and results of the competition can be found at <http://www.math.iit.edu/~weeklyproblem>  
You have to submit the solution by email, to [weeklyproblem@math.iit.edu](mailto:weeklyproblem@math.iit.edu)  
Please feel free to tell any IIT undergraduate student about the competition.

**Thank you for your participation  
Good Luck !**