NSF/CBMS Regional Conference in the Mathematical Sciences

Department of Applied Mathematics

Illinois Institute of Technology

August 9 - 13 · 2010 · Chicago · USA

Recent Advances in the Numerical Approximation of Stochastic Partial Differential Equations

Principal Lecturer

Peter E. Kloeden

University of Frankfurt, Germany

Organizers: Jinqiao Duan, Igor Cialenco, Fred J. Hickernell

Financial Support

National Science Foundation

ILLINOIS INSTITUTE OF TECHNOLOGY

Program

All events will take place in Engineering One Building

Conference Center: Room 106

All lectures/talks: Room 104

Contact person: Ms. Gladys Collins, collinsg@iit.edu, +1(312)-567-8980

Monday, August 9

Chairman: Jinqiao Duan

9:00 – 9:15	Registration, Lobby
9:15 - 9:30	Opening of the conference, Room 104 Ali Cinar, Vice Provost for Research and Dean of the Graduate College, IIT
9:30 – 10:30	Peter Kloeden, *Lec 1*
10:30 – 11:00	
11:00 – 11:30	Coffee Break
11:30 - 12:30	Peter Kloeden, *Lec 2*
12:30 – 2:00	Lunch
2:00 – 3:00	Panagiotis Souganidis (University of Chicago, USA) *Fully nonlinear stochastic PDE*
3:00 – 4:00	Tutorial *Introduction to numerical simulations for Stochastic ODEs*

Tuesday, August 10

Chairman: Fred J Hickernell

9:00 – 10:00	Peter Kloeden, *Lec 3*
10:00 - 10:30	Coffee Break
10:30 - 11:30	Peter Kloeden, *Lec 4*
11:30 - 12:30	Klaus Ritter (Darmstadt University, Germany) *Complexity Results for S(P)DEs*
12:30 - 2:00	Lunch
2:00 - 3:00	Arnulf Jentzen (Bielefeld University, Germany)
3:00 – 4:00	Break down sessions *Topic: General discussion on current research topics, developments and importance of numerical solutions for stochastic processes* Group 1, Room 104, Moderator Klaus Ritter Group 2, Room 124, Moderator Richard Sowers

Wednesday, August 11

Chairman: Igor Cialenco

9:00 – 10:00	Peter Kloeden, *Lec 5*
10:00 - 10:30	Coffee Break
10:30 - 11:30	Peter Kloeden, *Lec 6*
11:30 - 12:30	Anthony Roberts (Adelaide University, Australia) *Different coupling of finite elements discretises SPDEs*
12:30 - 2:00	Lunch
2:00 - 3:00	Salah Mohemmed (Southern Illinois University, USA) *Invariant Manifolds for Stochastic 2-D Navier Stokes Equations*
3:00 – 5:00	Poster session (E1, Lobby) Chairman: Arnulf Jentzen
5:00 -	Banquet

Thursday, August 12

Chairman: Greg Fasshauer

9:00 – 10:00	Peter Kloeden, *Lec 7*
10:00 - 10:30	Coffee Break
10:30 - 11:30	Peter Kloeden, *Lec 8*
11:30 - 12:30	Richard Sowers (University of Illinois at Urbana-Champaign, USA) *Stochastic Moving Boundary Value Problems*
12:30 - 2:00	Lunch
2:00 - 3:00	Arnulf Jentzen (Bielefeld University, Germany)
3:00 – 4:00	Panel discussion *Moderator:* Anthony Roberts *Panelists:* Hassan Allouba, Salah Mohemmed, Klaus Ritter, Panagiotis Souganidis, Richard Sowers

Friday, August 13

Chairman: Xiaofan Li

9:00 – 10:00	Peter Kloeden, Lec 9
10:00 - 10:30	Coffee Break
10:30 - 11:30	Peter Kloeden, Lec 10
11:30 - 12:30	Hassan Allouba (Kent State University, USA) *The Brownian-time approach to fourth order PDEs, SPDEs, SIEs, and SFPDEs*
12:30 - 2:00	Lunch
2:00 - 3:00	Break down sessions *Topic: Future research directions in Numerical Approximation of Stochastic Processes* Group 1, Room 104, Moderator Peter Kloeden Group 2, Room 124, Moderator Jinqiao Duan

Abstracts

Hassan Allouba

Kent State University

alllouba@math.kent.edu

The Brownian-time approach to fourth order PDEs, SPDEs, SIEs, and SFPDEs

We start by recalling our Brownian-time processes (BTPs) definition and connections to unconventional fourth order PDEs (and their equivalent fractional PDEs) as well as to Kuramoto-Sivashinsky (KS) type PDEs in all spatial dimensions, highlighting several important features (both probabilistically and from a PDEs standpoint).

We then move to our more recent papers concerning the stochastic equations part of our BTP program dealing with the new BTP stochastic integral equation (SIE) and KS-type SPDEs driven by space-time white noise in spatial dimensions $1 \le d \le 3$. Among the interesting results here is the compelling regularizing effect of these BTP kernels (like the density of Brownian-time Brownian motion) on their space-time white noise driven stochastic equations, which results in ultra regular L^p-bounded (locally in time) solutions in $1 \le d \le 3$. In space, these solutions are remarkably nearly locally Lipschitz for $d = 1, 2$; and nearly locally Hölder 1/2 in $d = 3$. In time, our solutions are locally γ-Hölder continuous with exponent $\gamma \in (0, \frac{4-d}{8})$ for $1 \le d \le 3$. We also discuss the spatial lattice multi-scale version of our BTP SIE and its role in proving the existence under less than Lipschitz condition on the diffusion coefficient of the BTP

SIE. This lattice model naturally led to our introduction of Brownian-time chains (the discrete version of BTPs)—of which Brownian-time random walk is a special case—and their corresponding fourth order differential-difference equations. The KS-type SPDEs have similar regularity results due to the intimate connection between the kernels in the BTP SIE and those KS SPDEs.

These new BTP SIEs and KS-type SPDEs results, using our explicit BTP representations approach, are at the core of several ongoing projects—at different stages of completion and with collaborations with several colleagues— dealing with qualitative and asymptotic analysis of a large class of fourth order and stochastic fractional PDEs. Time permitting, I'll briefly discuss some of those.

Daniel Conus

University of Utah

conus@math.utah.edu

Ito-Taylor expansions for the solution to non-linear stochastic parabolic and hyperbolic spde's

Using Walsh's stochastic integral, we study properties of multiple integrals. With this tool, we develop finite order Ito-Taylor expansions for the solution of non-linear parabolic and hyperbolic spde's. In some cases, convergence to the solution is conjectured as the order increases. Some ideas for the proof are presented. Joint work with Prof. Robert C. Dalang (EPFL, Lausanne)

Nathan Glatt-Holtz

Indiana University

negh@indiana.edu

Parameter Estimation for Nonlinear Stochastic Partial Differential Equations

Suppose we are attempting to describe a system with a model that depends on one or several parameters. Such parameters typically represent physical quantities we would like to measure. It is therefore desirable to develop 'in- verse' methods to accurately calibrate these parameters by observing incomplete or noisy data. On the other hand if the parameters are already known but we lack confidence in the underlying model such 'inverse' methods represent a means of testing and validating a model. Not surprisingly 'parameter estimation' problems arise frequently in applications and represent an important direction for the analysis of both deterministic and random dynamical systems. Given in particular the growing significance of nonlinear stochastic partial differential equations (SPDE) in modeling there is a clear need to develop the theory of parameter estimation for such systems. In this talk we discuss recent work in this direction for the stochastic Navier-Stokes equations. We consider the problem of determining the viscosity ν from the observation of a 2D fluid. Using the Girsanov theorem we derive several estimators for ν based on the first N Fourier modes of a single sample path observed on a finite time interval. We study the consistency and asymptotic normality of these estimators. The analysis treats strong, pathwise solutions for both the periodic and bounded domain cases in the presence of an additive white (in time) noise. This talk is based on recent joint work with Igor Cialenco.

Arnulf Jentzen

Bielefeld University, Germany

jentzen@math.uni-bielefeld.de

Talk 1: *On the Global Lipschitz Assumption in Computational Stochastics*

Stochastic differential equations are often simulated with the Monte Carlo Euler method. Convergence of this method is well understood in the case of globally Lipschitz continuous coefficients of the stochastic differential equation. The important case of superlinearly growing coefficients, however, remained an open question for a long time now. In this talk we overcome this difficulty and establish convergence of the Monte Carlo Euler method for a large class of one-dimensional stochastic differential equations whose drift functions have at most polynomial growth. The results in this talk are based on joint works with Martin Hutzenthaler, Peter E. Kloeden and Michael Roeckner.

Talk 2: *Taylor Expansions and Numerical Approximations for Stochastic Partial Differential Equations*

In this talk Taylor expansions for the solution process of a stochastic partial differential equation (SPDE) of evolutionary type are presented. Two numerical schemes for SPDEs with additive and non-additive noise respectively are derived on the basis of these Taylor expansions. The key advantage of the two numerical methods is that they break the computational complexity (number of computational operations and random variables needed to the compute the scheme) in comparison to previously considered algorithms for simulating SPDEs with additive space-time white and non-additive trace class noise respectively. The results in this talk are based on joint works with Peter E. Kloeden and Michael Roeckner.

Chia Ying Lee

Brown University

Chia_Ying_Lee@brown.edu

A Stochastic Finite Element Method for Stochastic Parabolic Equations Driven by Purely Spatial Noise

We consider parabolic SPDEs driven by purely spatial noise, and show the existence of solutions with random initial data and forcing terms. We perform error analysis for the semi-discrete stochastic finite element method applied to a class of equations with self-adjoint differential operators that are independent of time. The analysis employs the formal stochastic adjoint problem and the corresponding elliptic error estimates to obtain the optimal order of convergence (in space).

Joint of work with Boris Rozovskii

Salah Mohammed

Southern Illinois University

salah@sfde.math.siu.edu

Invariant Manifolds for Stochastic 2-D Navier Stokes Equations

We study the dynamics of a two-dimensional stochastic Navier-Stokes equation on a smooth bounded domain, driven by affine (linear + additive) white noise. We show that solutions of the 2D Navier-Stokes equation generate a locally compacting C^1 semiflow. Using ergodic theory and dynamical systems techniques, we establish a stable manifold theorem for a hyperbolic equilibrium. We also show the existence of local and global invariant foliations of the energy space when the equilibrium is ergodic.

Sivaditya Kaligotla

University of Southern California

kaligotl@usc.edu

Wick Product in the Stochastic Burgers Equation: A Curse or a Cure?

It has been known for a while that a nonlinear equation driven by singular noise must be interpreted in the re-normalized, or Wick, form. For the stochastic Burgers equation, Wick nonlinearity forces the solution to be a generalized process no matter how regular the random perturbation is, whence the curse. On the other hand, certain multiplicative random perturbations of the deterministic Burgers equation can only be interpreted in the Wick form, whence the cure. The analysis is based on the study of the coefficients of the chaos expansion of the solution at different stochastic scales.

Klaus Ritter

TU Kaiserslautern, Germany

`ritter@mathematik.tu-darmstadt.de`

Complexity Results for S(P)DEs

Stochastic differential and stochastic partial differential equations (SDEs and SPDEs, resp.) give raise to various computational problems like strong and weak approximation as well as cubature.

In this talk we will discuss some of these problems within the framework of information-based complexity (IBC). This framework allows to study the following basic question: what is the minimal error $e(n)$ that can be achieved by any algorithm with cost at most n. The sequence of minimal errors then quantifies the computational difficulty of the computational problem at hand, and it serves as a benchmark for the construction of new algorithms.

First we provide an introduction to IBC and then we discuss specific complexity results for SDEs and SPDEs.

Tony Roberts

University of Adelaide, South Australia

`anthony.roberts@adelaide.edu.au`

Different coupling of finite elements discretises SPDEs

Stochastic centre manifold theory provides novel support for coarse grained, macroscale, spatial discretisations of nonlinear stochastic partial differential equations such as the example of the stochastically forced Burgers' equation. Dividing the physical domain into finite length overlapping elements empowers the approach to resolve fully coupled dynamical interactions between neighbouring elements. I explore, compare and contrast different methods for coupling the finite elements. The techniques developed here may be applied to discretise many dissipative stochastic partial differential and difference equations.

Panagiotis Souganidis

University of Chicago

`souganidis@math.uchicago.edu`

Fully nonlinear stochastic PDE

I will present a review and recent developments of the theory of stochastic viscosity solutions for fully nonlinear first- and second-order stochastic partial differential equations.

Richard Sowers

University of Illinois at Urbana-Champaign

`r-sowers@illinois.edu`

Stochastic Moving Boundary Value Problems

We discuss the effect of noise on moving boundary value problems. Moving boundary value problems present unique theoretical and unique challenges. Our interest is to see how stochasticity affects these types of problems. We consider several one-dimensional problems and several types of noise. Namely, a problem in flame propagation and the Stefan problem, and we consider Brownian and spatially correlated noise. We understand how to pose these problems in a tractable way, and discuss some theory and algorithms.

Shengqiang Xu
Illinois Institute of Technology
sxu9@iit.edu

A Taylor Expansion Approach for Solving Partial Differential Equations with Random Neumann Boundary Conditions

Nonlinear partial differential equation with random Neumann boundary conditions are considered. A stochastic Taylor expansion method is derived to simulate these stochastic systems numerically. As examples, a nonlinear parabolic equation (the real Ginzburg-Landau equation) and a nonlinear hyperbolic equation (the sine-Gordon equation) with random Neumann boundary conditions are solved numerically using a stochastic Taylor expansion method. The impact of boundary noise on the system evolution is also discussed.

Qi Ye
Illinois Institute of Technology
qye3@iit.edu

Approximation of Linear Stochastic Partial Differential Equations by Gaussian Processes via Matérn Functions

We have a discrete data with independent noises

$$\begin{aligned}
&X_{\mathcal{D}} = \{\boldsymbol{x}_1, \cdots, \boldsymbol{x}_N\} \subset \mathcal{D}, \quad X_{\partial\mathcal{D}} = \{\boldsymbol{x}_{N+1}, \cdots, \boldsymbol{x}_{N+M}\} \subset \partial\mathcal{D},\\
&\boldsymbol{Y}_{\mathcal{D},k} = f_k(X_{\mathcal{D}}) + \xi_{k,X_{\mathcal{D}}}, \quad \xi_{k,\boldsymbol{x}} \sim \mathcal{N}(0, \sigma_{k,f}(\boldsymbol{x})^2),\\
&\boldsymbol{Y}_{\partial\mathcal{D},l} = g_l(X_{\partial\mathcal{D}}) + \zeta_{l,X_{\partial\mathcal{D}}}, \quad \zeta_{l,\boldsymbol{x}} \sim \mathcal{N}(0, \sigma_{l,g}(\boldsymbol{x})^2),
\end{aligned}$$

generated from a linear elliptic partial differential equations model

$$\begin{cases}
L_1 u(\boldsymbol{x}) = f_1(\boldsymbol{x}), \cdots, L_n u(\boldsymbol{x}) = f_n(\boldsymbol{x}), & \boldsymbol{x} \in \mathcal{D},\\
B_1 u(\boldsymbol{x}) = g_1(\boldsymbol{x}), \cdots, B_m u(\boldsymbol{x}) = g_m(\boldsymbol{x}), & \boldsymbol{x} \in \partial\mathcal{D},
\end{cases}$$

where $\mathcal{D}$ is a regular bounded domain of $\mathbb{R}^d$ and

$$L_k = \sum_{|\alpha| \le n_l} c_{k,\alpha} D^\alpha|_{\mathcal{D}}, \ c_{k,\alpha} \in \mathrm{C}(\overline{\mathcal{D}}), \quad B_l = \sum_{|\alpha| \le n_b} b_{l,\alpha} D^\alpha|_{\partial\mathcal{D}}, \ b_{l,\alpha} \in \mathrm{C}(\partial\mathcal{D}).$$

Here we will approximate the function value $u(\boldsymbol{x})$ by a normal random variable $U_{\boldsymbol{x}}$ conditioned on the given data, i.e.,

$$u(\boldsymbol{x}) \approx U_{\boldsymbol{x}} \sim \mathcal{N}(\hat{\mu}(\boldsymbol{x}), \hat{\sigma}(\boldsymbol{x})^2), \quad \boldsymbol{x} \in \mathcal{D},$$

where the mean $\hat{\mu}(\boldsymbol{x})$ and the variance $\hat{\sigma}(\boldsymbol{x})$ can be computed by the data and a covariance kernel function K (Matérn function) with respect to the linear operators L_k and B_l. A Gaussian process (Gaussian field) is firstly constructed on a reproducing-kernel Hilbert space $\mathrm{H}_K(\mathcal{D})$ corresponding to the Matérn function. It means that $\mathrm{H}_K(\mathcal{D})$ is seen as a sample space Ω of a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Combining this with a Bayesian approach, we are able to obtain confidence interval for $u(x)$ (instead of an explicit approximating solution) through a covariance matrix for the above differential operators L_k and boundary operators B_l on the data.