Recitation problems for Tuesday Apr 20, 11:25am-12:15pm

1. Alon and Spencer p.62 #5 (p.57 in 2nd ed.); Hard: #2, #3 on the same page. (1 problem = 1 recitation credit of your needed 3.)

2. Prove the details of the asymptotic lower bound of $R(k,k) > \frac{\sqrt{2}}{e}(1+o(1))k2^{k/2}$ obtained from the bound
$$e\left(\binom{k}{2}\binom{n}{k-2}+1\right)\cdot 2^{1-\binom{k}{2}} < 1 \quad \Rightarrow \quad R(k,k) > n$$
given by application of the Lovász Local Lemma.

3. (Medium) Compute the expected value of z^* in the improved matching algorithm presented in lecture 10. Convincing evidence presented with a computer algebra system is acceptable. The derivation may include some subtle conditional probabilities. Be sure to make a clean presentation.

4. Let G be a simple graph. Define $b(G)$ to be the maximum number of edges in a bipartite subgraph of G. For a fixed $v \in V(G)$, recall that $N(v)$ is the neighborhood of v. Each $N(v)$ yields a bipartite subgraph of G in a natural way (for you to determine). Use the Cauchy-Schwarz inequality to show that
$$b(G) \geq \frac{4e^2}{n^2} - \frac{6t}{n}.$$
Hints: every triangle can be viewed as coming from the neighborhood of any one of its vertices; try averaging over the vertices.

5. Apply the alteration/deletion method to a question pertaining to your own research. You must convince the audience that the question you address is reasonable, but do not necessarily have to do a complex calculation.

6. Let $W(k)$ be the least integer n such that for any two-coloring of $\{1,2,...,n\}$, there is a monochromatic arithmetic progression of length k.
(a) Use the basic probabilistic method to show that $W(k) \geq 2^{k/2}$.
(b) Use the Lovasz Local Lemma to show that $W(k) > \frac{1}{2e}\frac{2^k}{k}(1+o(1))$.

7. Concisely and clearly present solutions to HW4 #2 or HW5 #1.