Homework 6

Written problems for Tuesday, Apr 23rd, 2010

Turn in at the beginning of class, or in my mailbox at any time before class. Don't forget to read the homework collaboration and use of references policy on the first day handout if you have not already.

These questions refer to the lecture on "Guessing Secrets" on April 15, 2010. The space of secrets is $\Omega = \{0,1\}^n$. Paul's questions are of the form $F : \Omega \to \{0,1\}$, and Carole responds with either 0 or 1. Carole has two secrets $\{x_1, x_2\} \subseteq \Omega$, and she must answer either $F(x_1)$ or $F(x_2)$. We assume a general adversarial viewpoint in which Carole does not have to pick $\{x_1, x_2\}$ beforehand, but must answer so that at the end of all questions, there is at least one candidate 2-set for $\{x_1, x_2\}$. We call the initial graph on which the progress of the game is encoded K_Ω, the complete graph whose vertices are labeled by the elements of Ω.

1. Let $H \subset K_\Omega$ be a fixed subgraph. Fix a question strategy for Paul in the $k = 2$ guessing secrets game. Regardless of what strategy Paul chooses, prove that Carole can preserve $H \subseteq K_\Omega$ iff the edges of H form a nonempty intersecting family.
2. Prove that every nonempty intersecting family of edges of a (simple) graph is either a triangle or a star.
3. Use the first moment method to prove the existence of an oblivious winning strategy for Paul of at most
$$\left\lceil \frac{4}{\log_2(8/7)} n \right\rceil \tag{1}$$
questions.
4. (5 points extra credit.) Now suppose we wish to write down an explicit winning strategy of the length (number of questions) given by (1). Assuming full independence, what is the size of the probability space that must be searched? Can we reduce the number of independent coin flips needed to find a winning strategy of the length in (1) in a way similar to that of HW3 #4? (This is not an easy question.)