PRINT Last name:____________________ First name:____________________

Signature:____________________ Student ID:____________________

Math 475 Exam 3, Fall 2007

Instructions. You must show work in order to receive full credit. Partial credit is possible for work which makes positive progress toward the solution.

Conditions. ***No calculators***, notes, books, or scratch paper. By writing your name on the exam you certify that all work is your own, under penalty of all remedies outlined in the student handbook. Please do not talk until after leaving the room.

Time limit: 1 hour 15 minutes (strict).

NOTE: The topics may not be in order either of increasing difficulty or of the order they were covered in the course. Answers (sums/products/quotients of fractions or binomial coefficients) **do not need to be fully simplified**, nor do you need to find numerical approximations for your answers.

Distribution	Probability function	Mean	Variance	MGF
Uniform				
Normal	$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(y-\mu)^2}{2\sigma^2}\right]$	μ	σ^2	$\exp\left(\mu t + \frac{t^2\sigma^2}{2}\right)$
Exponential	$f(y) = \frac{1}{\beta} e^{-y/\beta}$	β	β^2	$(1-\beta t)^{-1}$
Gamma	$f(y) = \left[\frac{1}{\Gamma(\alpha)\beta^\alpha}\right] y^{\alpha-1} e^{-y/\beta}$	$\alpha\beta$	$\alpha\beta^2$	$(1-\beta t)^{-\alpha}$
Hypergeometric	$p(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}$	$\frac{nr}{N}$	$n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$	
Binomial				
Geometric		$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$
Poisson	$p(y) = \frac{\lambda^y}{y!} e^{-\lambda}$			
Negative binomial	$p(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r}$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{pe^t}{1-(1-p)e^t}\right]^r$

$$p(y_1 \mid y_2) = \frac{p(y_1, y_2)}{p_2(y_2)}$$

Tchebysheff: $P(|Y-\mu| \geq k\sigma) \leq \frac{1}{k^2}$

SHOW WORK FOR FULL CREDIT NO CALCULATORS

1. (16pts) Let Y_1 and Y_2 be continuous random variables with joint density

$$f(y_1, y_2) = \begin{cases} \frac{2}{3}, & 0 \le y_1 \le 3, \quad 0 \le y_2 \le 1, \quad 3y_2 \le y_1 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Sketch the region of support of $f(y_1, y_2)$ (where $f(y_1, y_2) > 0$).
(b) Compute the marginal probability densities $f_1(y_1)$ and $f_2(y_2)$.
(c) Compute the conditional probability density of Y_1 given that $Y_2 = \frac{1}{3}$.
(d) Compute $\mathrm{P}(Y_2 \le 1/2 \mid Y_1 \le 2)$.
(e) Are Y_1 and Y_2 independent? Briefly explain why or why not.

2. Assume that X and Y are random variables with

$$\begin{aligned} E(X) &= 3 & V(X) &= 4 \\ E(Y) &= -1 & V(Y) &= 2 \\ E(XY) &= -1\,. \end{aligned}$$

(a) Compute $E(3X - 5Y)$.
(b) Compute $V(3X - 5Y)$.
(c) Are X and Y independent? Briefly explain why or why not.

3. Let Y be a continuous random variable with density function

$$f_Y(y) = \begin{cases} \dfrac{3}{y^2}, & y > 3 \\ 0, & \text{otherwise.} \end{cases}$$

Define $U = 5\ln(Y - 3)$.
(a) Compute the density function $f_U(u)$ for U.
(b) Give the interval of support of U.

4. Let $Y_1, Y_2, \ldots, Y_{25}$ be independent random variables each with $E(Y_i) = \mu$ and $V(Y_i) = \sigma^2$. They represent 25 repeated independent samples from the same population. The function

$$\overline{Y} := \frac{Y_1 + Y_2 + \cdots + Y_{25}}{25}$$

is called the *sample mean* of the population. Use Tschebycheff's Theorem to find the probability that $\overline{Y}$ is within $\sigma/2$ of $E(\overline{Y})$.

5. Three radioactive atoms are observed until they decay. The individual times of decay are Y_1, Y_2, and Y_3, respectively, which are independent and exponentially distributed, each with mean 12 (seconds). Define $U = Y_1 + Y_2 + Y_3$.
(a) Compute $E(U)$ by finding the moment generating function for U in terms of those of Y_1, Y_2, and Y_3, and then identifying the mean of the resulting distribution.
(b) Compute $E(U)$ using properties of expected value.

6. Y_1 and Y_2 are discrete random variables whose joint distribution $p(y_1, y_2)$ is given in the table.
(a) Determine the marginal probability functions $p_1(y_1)$ and $p_2(y_2)$.
(b) Compute $P(Y_1 \geq 2 \,|\, Y_2 = 1)$.
(c) Are Y_1 and Y_2 independent? If yes, explain why. If not, re-define $p(y_1, y_2)$ so that Y_1 and Y_2 are independent but have the same marginal probability functions as in **(a)**.

		Y_1			
	$p(y_1, y_2)$	1	2	3	
Y_2	1	.1	.2	.1	
	2	.2	.3	.1	

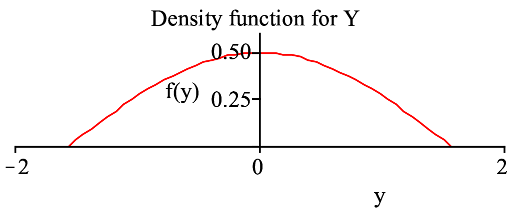
7. On a typical day, a store sells a fraction Y_1 of its stock of milk and a fraction Y_2 of its stock of soda. The store's revenue from these sales is $R = 300Y_1 + 200Y_2$. The joint density function for Y_1 and Y_2 is

$$f(y_1, y_2) = \begin{cases} 1 + y_1 - y_2, & 0 \le y_1 \le 1, \quad 0 \le y_2 \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Compute $E(R)$.
(b) Are Y_1 and Y_2 independent? Briefly explain why or why not.

8. Assume Y is a continuous random variable with joint density function

$$f_Y(y) = \begin{cases} \frac{1}{2}\cos(y), & -\pi/2 \le y \le \pi/2 \\ 0, & \text{otherwise.} \end{cases}$$



Let $U = Y^2$. (Not 1-1 on the support of Y.)
(a) Compute the density $f_U(u)$ of U.
(b) Give the interval on which U is supported.