

PRINT Last name: _____ First name: _____

Signature: _____ Student ID: _____

Math 475 Exam 1, Fall 2007

Instructions. You must show work in order to receive full credit. Partial credit is possible for work which makes positive progress toward the solution.

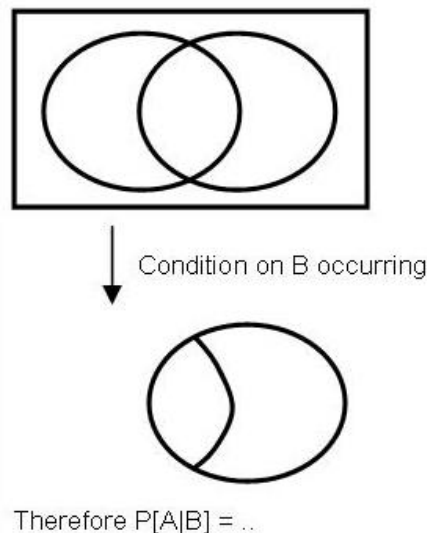
Conditions. *No calculators*, notes, books, or scratch paper. By writing your name on the exam you certify that all work is your own, under penalty of all remedies outlined in the student handbook. Please do not talk until after leaving the room.

Time limit: 1 hour 15 minutes (strict).

NOTE: The topics may not be in order either of increasing difficulty or of the order they were covered in the course.

Keep the exam closed until directed.

Meanwhile, you could complete the following (not graded):



$$\text{Also helpful: } P(B_j|A) = \frac{P(A|B_j)P(B_j)}{\sum_i P(A|B_i)P(B_i)}$$

SHOW WORK FOR FULL CREDIT

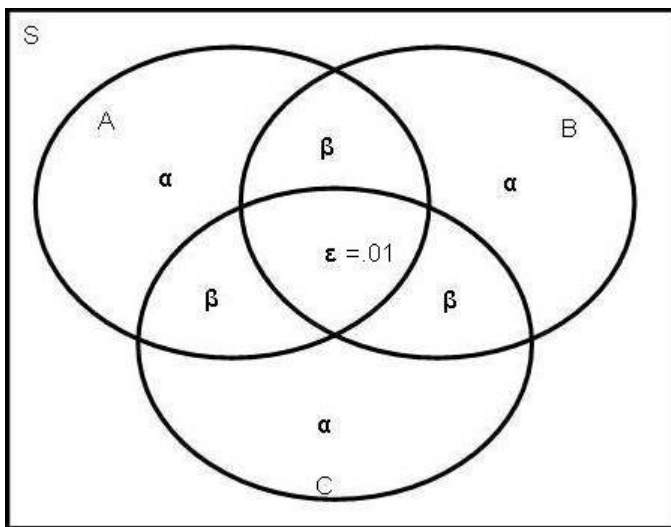
NO CALCULATORS

1. An experiment consists of flipping a fair coin and rolling a 6-sided die with the following properties: (i) each even die outcome is equally likely, (ii) each odd die outcome is equally likely, and (iii) an even die outcome is exactly twice as likely as an odd die outcome. The outcome of the coin is independent of the outcome of the die.
 - (a) List all of the sample points (simple events) of the experiment along with their probabilities.
 - (b) What is the probability that the the coin is tails and the die roll is ≥ 4 ?
 - (c) Given that the die roll is odd, what is the probability that the coin toss is heads?

2. A coin is flipped twice in a row. The probability that the first toss is heads is $1/2$. The probability that the second toss is heads is also $1/2$. However, The probability that both tosses are heads is $1/6$, and the probability that both tosses are tails is $1/6$ (perhaps due to some mysterious quantum coupling).
 - (a) Label each of the four events to which probabilities have been assigned in the problem statement.
 - (b) Write each sample point of the experiment in terms of the events in (a).
 - (c) Determine the probability that the first coin flip is tails and the second coin flip is heads.

3. Illustrate the following sets on three *separate* Venn diagrams: $A \cap B$, $\bar{A} \cap \bar{B}$, and $(A \cap B) \cup (\bar{A} \cap \bar{B})$. **Label** each Venn diagram with the set it represents.

4. Events A_1, \dots, A_n are *k-wise independent* provided $P(A_{i_1} \cap \dots \cap A_{i_k}) = P(A_{i_1}) \dots P(A_{i_k})$ for any choice of k distinct events (or their complements) from the original n events.
- (a) Design a probability space by finding α and β in the appropriate sections of the Venn diagram so that A , B , and C are 2-wise independent but not 3-wise independent. (For example, $P(A \cap B \cap \bar{C}) = \beta$.) You are given that $P(A \cap B \cap C) = \epsilon = .01$.
- (b) Compute $P(A)$, $P(A \cap B)$, and $P(A \cap B \cap C)$ given your choices of α and β in (a).



5. Bowl 1 contains 20 plain cookies and 20 chocolate chip cookies. Bowl 2 contains 10 plain cookies and 30 chocolate chip cookies. An experiment is conducted as follows: (i) a bowl is selected at random, with either being equally probable, and (ii) a cookie from within the chosen bowl is selected at random with each being equally probable.
- (a) What is the probability that a chocolate chip cookie is selected given that Bowl 2 is selected?
 - (b) What is the probability that a chocolate chip cookie is selected?
 - (c) What is the probability that Bowl 2 was selected given that a chocolate chip cookie has been selected?

6. A package courier has 4 packages which each have a distinct destination. Due to a malfunction at the office, the destinations are scrambled, so that each package still has exactly one destination, but each way of assigning packages to destinations is equally likely. What is the probability that exactly 2 packages will reach their correct destinations?

7. A deck of cards consists of 52 cards, represented by the product set $\{2, \dots, 10, J, Q, K, A\} \times \{C, D, H, S\}$. That is, each card has one of 13 *face values* and one of 4 *suits*. The deck is shuffled to be completely random, and cards are dealt so that (i) Player 1 receives 4 cards, (ii) Player 2 receives 4 cards, and (iii) 2 cards are placed on the table. The remaining cards are left unused.
- (a) What is the probability that Player 1 is dealt 4 cards with the same face value?
 - (b) Given that the two cards on the table are $(2, C)$ and $(4, H)$, what is the probability that Player 1 has been dealt 4 cards with the same face value?
 - (c) What is the probability that Player 1 is dealt 4 cards with the same suit?
- (It is unnecessary to completely simplify your answers.)*

8. Medical case histories indicate that different illnesses may produce identical symptoms. Suppose that a particular set of symptoms, denoted H , occurs only when any one of three illnesses I_1 , I_2 , or I_3 , occurs. Assume that the simultaneous occurrence of more than one of these illnesses is impossible and that

$$P(I_1) = .05, \quad P(I_2) = .001, \quad \text{and} \quad P(I_3) = .02.$$

The probabilities of developing the set of symptoms H , given each of the illnesses, are known to be

$$P(H|I_1) = .90, \quad P(H|I_2) = .95, \quad \text{and} \quad P(H|I_3) = .75.$$

- (a) If a person is selected at random, what is the probability that the person exhibits the symptoms H ?
- (b) Select a person at random. Given that the person exhibits the symptoms H , what is the probability that the person has illness I_1 ?