Chapter 21 Random Quiz

(\text{Let } a \in E, \text{ where } E \text{ is an extension field of } F.\text{)}

1. a is algebraic over F provided \quad (\text{fill in definition})

2. Suppose a is transcendental over F. What field is \( F(a) \) isomorphic to? (express without 'a').

3. Given that \( \mathbb{Q}(\sqrt[3]{2}) \) has dimension 3 over \( \mathbb{Q} \), and \( \mathbb{Q}(\sqrt[3]{2}, \sqrt[3]{-1}) \) has dimension 2 over \( \mathbb{Q}(\sqrt[3]{2}) \), what is the dimension of \( \mathbb{Q}(\sqrt[3]{2}, \sqrt[3]{-1}) \) over \( \mathbb{Q} \)? (Interpret \( \sqrt[3]{-1} \) as a primitive 3rd root of unity.)

4. Name a field and an element that is transcendental over that field,