Structure of Fund Thm

Lemma 1 Express $G = H \times K$ $(|G| = p^n m)$
where $|H| = p^n$, $|K| = m$, $p \nmid m$.

Lemma 2 Express $H = \langle a \rangle \times H'$,
$|\langle a \rangle|$ maximum

Lemma 3 Induction on $|H|$ to get

$$= \langle a_1 \rangle \times \langle a_2 \rangle \times \cdots \times \langle a_g \rangle$$

Lemma 4 Any two two prime-power order cyclic decompositions are the same up to isomorphism of individual cyclic factors.

Decomposition for Abelian G, $|G| = p^n$

1. Compute $|x|$ for all $x \in G$.
2. Select $a_i \in G$ with $|a_i|$ maximum.
   Define $G_1 = \langle a_1 \rangle$.
   Set $i = 1$.
3. If $|G| = |G_i|$, stop. Otherwise, $i \leftarrow i+1$.
4. Select $a_i \in G$ with $|a_i|$ maximum such that
   - $|a_i| = p^k$
   - $p^k \leq |G| / |G_{i-1}|$
   - none of $a_i, a_i^p, a_i^{p^2}, \ldots, a_i^{p^{k-1}} \in G_{i-1}$,

   and define $G_i = G_{i-1} \times \langle a_i \rangle$.
5. Return to 3.

We only have to check $a_i, a_i^p, a_i^{p^2}, \ldots, a_i^{p^{k-1}}$ because ...