Group Members: ____

(1) Without referring to the book, prove Theorem 10.2 Part 4: Kerϕ is a subgroup of G when $\phi : G \to \overline{G}$ is a homomorphism. Use the 1-step or 2-step subgroup test.

(2) Prove the reverse implication in Part 5 of Theorem 10.1. That is, $a\text{Ker}\phi = b\text{Ker}\phi \Rightarrow \phi(a) = \phi(b)$ for any homomorphism $\phi : G \to \overline{G}$. Part 4 on page 138 will be useful.

(3) Without referring to the book, prove Theorem 10.2.3, that is, if H is Abelian, then $\phi(H)$ is Abelian.

(4) Without referring to the book, prove Theorem 10.2.7, that is, if $\overline{K} \leq \overline{G}$, then $\phi^{-1}(K) \leq G$.

(5) Define the homomorphism $\phi : D_4 \to \mathbb{Z}_{10}$ by sending all rotations to 0 and letting $\phi(H) = 5$.
(a) Compute $G/\text{Ker}\phi$ and $\phi(G)$.
(b) Is $G/\text{Ker}\phi$ a group? Why or why not?
(c) Are $G/\text{Ker}\phi$ and $\phi(G)$ the same size? Isomorphic? If so give an isomorphism.

(6) Define the homomorphism $\phi : D_4 \to \mathbb{Z}_2 \oplus \mathbb{Z}_2$ by $\phi(R_0) = \phi(R_{180}) = (0, 0)$, $\phi(R_{90}) = (1, 0)$, and $\phi(D) = (0, 1)$.
(a) Finish the definition of ϕ for all elements of D_4.
(b) Compute $G/\text{Ker}\phi$ and $\phi(G)$.
(c) Is $G/\text{Ker}\phi$ a group? Why or why not?
(d) Are $G/\text{Ker}\phi$ and $\phi(G)$ the same size? Isomorphic? If so give an isomorphism.

Theorem 10.3: First Isomorphism Theorem
Let $\phi : G \to \overline{G}$ be a group homomorphism. Then the mapping from $G/\text{Ker}\phi$ to $\phi(G)$, given by $g\text{Ker}\phi \to \phi(g)$, is an isomorphism, so that $G/\text{Ker}\phi \approx \phi(G)$.

Corollary
If $\phi : G \to \overline{G}$ is a homomorphism, then $|\phi(G)|$ divides both $|G|$ and $|\overline{G}|$.
