Group Members: ____

Ex. 7: Index 2 Subgroups are Normal
Let $|G|$ be a group and $H \leq G$ such that $|G : H| = 2$. Then $H \lhd G$.

(1) Prove the above theorem using the following steps.
(a) Write the form of the two left cosets of H in G as H, aH.
(b) Draw the simple diagram of the partition of G into these left cosets. What must be true about a to guarantee that $H \neq aH$?
(c) Now consider H, Ha. Use the fact you observed about a to conclude that $H \neq Ha$. Why are these observations enough to conclude that $H \lhd G$? (Refer to p.139 if necessary.)

A_4 has no subgroup of order 6. (Second proof.)

Theorem 9.3: The G/Z Theorem
Let G be a group and let $Z(G)$ be the center of G. If $G/Z(G)$ is cyclic, then G is Abelian.

Theorem 9.4: $G/Z(G) \approx \text{Inn}(G)$
For any group G, $G/Z(G)$ is isomorphic to $\text{Inn}(G)$.

Theorem 9.5: Cauchy's Theorem for Abelian Groups
Let G be a finite Abelian group and let p be a prime that divides the order of G. Then G has an element of order p.

(2) Prove that if G is a non-Abelian group of order pq, where p, q are primes, then $|Z(G)| = 1$. Use the following steps.
(a) What does Lagrange's Theorem say about the possibilities for $|Z(G)|$?
(b) Which possibility can you immediately rule out from the hypothesis on G?
(c) Which possibility do you not have to check due to the conclusion of the statement?
(d) For the other possibilities, what is $|G/Z(G)|$? Apply Corollary 3 on p.142 and Theorem 9.3.

(3) Use the proof of Theorem 9.4 to directly compute $\text{Inn}(D_4)$ by partitioning D_4 into the left cosets of $Z(D_4)$ and selecting one representative of each coset. See p.33.