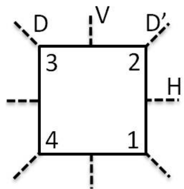
Group Members: ____

(1) This question involves converting group symmetries of D_4 into permutations.

(a) It is straightforward to see that D_4 is generated by R_{90}, and H, i.e., $D_4 = \langle R_{90}, H\rangle$. Write $\alpha = R_{90}$ and $\beta = H$ as permutations by observing the start and finish point of each number under the symmetry. Use two-line notation.

(b) Compute α^{-1}, β^{-1}, $\alpha\beta$, and $\beta\alpha$. Write down the previous name we used for each of these group elements (R_*, H, D, etc.).



(Continued on reverse)

(2) Rotational symmetries of the tetrahedron.

(a) By ignoring labels and handedness of rotation, there are 5 distinct types of plane symmetries of the square comprising the group D_4 (i.e., reflections and rotations): identity, 90 degree rotation (2 total), 180 degree rotation, reflection through a diagonal line (2 total), and reflection through a line containing midpoints of opposite sides (2 total). Similarly, how many different types of rotational symmetries of the regular tetrahedron are there (ignore reflections)?

(b) List the rotational symmetries of the tetrahedron (written as permutations on $\{A, B, C, D\}$, grouped together according to the types of rotations identified in (1).

(c) Is this group in (b) the same as S_4, the set of permutations on $\{1, 2, 3, 4\}$?

(d) Refer to Table 5.1 on p.107 to determine the centralizer of $(AC)(BD)$ (also written (13)(24)) from the Cayley Table. Visualize $C((AB)(CD))$ with your tetrahedron figure by verifying that $\gamma\alpha\gamma^{-1} = \alpha$ for every $\gamma \in C(\alpha)$. By just exploring with your tetrahedron model and without looking at the Cayley Table, find three elements that are *not* in the centralizer of (BCD). (This last skill might be tested on an exam.)