Group Members: ______________________________

(1) For this question, collect data about orders of elements and about powers of elements of cyclic groups, grouped according to when different powers yield the same element. The groups are $\mathbb{Z}_5$, $\mathbb{Z}_6$, and $\mathbb{Z}_8$.

a	a^1	a^2	a^3	a^4	a^5	$\lvert a \rvert$	sets of $\{i, j, k \ldots \mid a^i = a^j = a^k \cdots\}$
0							
1							
2							
3							
4							

a	a^1	a^2	a^3	a^4	a^5	a^6	$\lvert a \rvert$	sets of $\{i, j, k \ldots \mid a^i = a^j = a^k \cdots\}$
0								
1								
2								
3								
4								
5								

a	a^1	a^2	a^3	a^4	a^5	a^6	a^7	a^8	$\lvert a \rvert$	sets of $\{i, j, k \ldots \mid a^i = a^j = a^k \cdots\}$
0										
1										
2										
3										
4										
5										
6										
7										

(2a) Conjecture a condition for when $a \in \mathbb{Z}_n$ is a generator of $\mathbb{Z}_n$.

(2a) Conjecture a condition for when $a^i = a^j$ in $\mathbb{Z}_n$.

Break.

Theorem 4.1 Criterion for $a^i = a^j$. Let G be a group, and let $a \in G$. If $|a| = \infty$, then all distinct powers of a are distinct group elements. If $|a| < \infty$, say $|a| = n$, then $\langle a \rangle =$ ________________ and $a^i = a^j$ iff ____________.

Corollary 1 $|a| = |\langle a \rangle|$**.** For any group element a, $|a| = |\langle a \rangle|$.

Corollary 2 $a^k = e \Rightarrow |a||k$**.** Let G be a group and let $a \in G$ be an element of order n. If $a^k = e$, then n divides k.

(3) From Theorem 4.1, prove the following Corollary 1: For any group element a, $|a| = |\langle a \rangle|$. (Treat finite and infinite order cases separately.)

(4) From Theorem 4.1, prove the following Corollary 2: Let G be a group and let a be an element of order $n \in \mathbb{Z}^+$ in G. If $a^k = e$, then n divides k.