PRINT Last name:____________________ First name:____________________

Signature:____________________ Student ID:____________________

Math 430-01 Exam 1, Spring 2011

[Workspace]

I. Examples, Counterexamples and short answer. (6 pts ea.) Do not give proofs, but clearly indicate your proposed example or counterexample, or short answer where appropriate.

1. The identification scheme for a United States Postal Service money order consists of a 10-digit identification number plus one digit of check in the form

$$d_1\, d_2\, \cdots\, d_9\, d_{10} \,|\, c,$$

where the check digit is the entire 10-digit number mod 9. An error is detected when the check digit does not satisfy the check digit formula.

A certain money order has identification number 1823492514 with check $c = 3$.
(a) Give an example of a single-digit error in the identification number that is detected by the scheme.
(b) Give an example of a single-digit error in the identification number that is not detected by the scheme.

2. Suppose you have an infinite supply of postage stamps with the following denominations: 4, 5, and 10 cents. What is the largest postage value in cents that cannot be satisfied? (Do not write a proof.)

3. Draw one figure whose plane symmetries form a cyclic group, and a second figure whose plane symmetries form a dihedral group. Clearly label or distinguish them.

4. Write down the Cayley table for the Abelian group of order 4 that has no elements of order 4.

5. Write down a list of distinct subgroups G_1, G_2, G_3, G_4, G_5 such that G_1 and G_2 have finite order, G_4 and G_5 have infinite order, and $G_1 \leq G_2 \leq G_3 \leq G_4 \leq G_5$.

6. Write down examples of a group G and nonidentity elements $a, b, c \in G$ such that a and b commute but a and c do not commute.

7. Give an example of a group G and an element $a \in G$ such that $\{a^i \mid i \in \mathbb{Z}^+\} \neq \{a^i \mid i \in \mathbb{Z}\}$.

8. Give an example of a group $U(n)$ and a subgroup H of $U(n)$ such that the order of H is neither 1 nor $|U(n)|$.

II. Constructions and Algorithms. Do not write proofs, but do give clear, concise answers, including steps to algorithms where applicable.

9. (14 pts) For this question, the group G has Cayley Table as given.
(a) Find the center $Z(G)$.
(b) Find the centralizer $C(3)$.
(c) Find a proper subgroup of G that is as large as possible. If it helps, assume the following fact that we have not yet proved: the order of a subgroup must divide the order of the group.

G	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	5	4	7	6	1	8	3
3	3	8	5	2	7	4	1	6
4	4	3	6	5	8	7	2	1
5	5	6	7	8	1	2	3	4
6	6	1	8	3	2	5	4	7
7	7	4	1	6	3	8	5	2
8	8	7	2	1	4	3	6	5

10. (8 pts) Find $\gcd(233, 55)$ using the Euclidean algorithm. Find $s, t \in \mathbb{Z}$ such that $\gcd(233, 55) = 233 \cdot s + 55 \cdot t$.

III. Proofs. (10 pts ea.) Part of the score is determined by careful formatting of the proof (forward and reverse directions, assumptions, conclusions, stating whether you are proving by direct proof, contrapositive, contradiction, induction, etc.). Partial credit will be awarded for this as well.

12. For every positive integer n prove that 3 divides $4^n - 1$.

13. Suppose that a and b are group elements and that $a^{-1}ba = b^{-1}$ and $b^{-1}ab = a^{-1}$. Prove that $a^2 = b^{-2}$

Prove ***ONE*** out of 14-15. Clearly indicate which proof you want graded.

14. Let G be a group and let $a \in G$. Prove that the center $Z(G)$ of G is a subgroup of the centralizer $C(a)$ of a.

15. Let $\mathbb{R}^*$ be the group of nonzero real numbers under multiplication, let $d \in \mathbb{Z}^+$ be a positive integer, and let $H = \{x \in \mathbb{R}^* \mid x^d \text{ is rational}\}$. Prove that H is a subgroup of $\mathbb{R}^*$.