Lagrange’s Theorem: $|H|$ divides $|G|$
If $G$ is a finite group and $H$ is a subgroup of $G$, the $|H|$ divides $|G|$. Moreover, the number of distinct left (right) cosets of $H$ in $G$ is $|G|/|H|$. 

Definition: Index of $H$ in $G$
The index of $H$ in $G$ is written as $|G : H|$ and defined to be the number of cosets of $H$ in $G$.

**Corollary 1 of Lagrange’s Theorem**
If $G$ is a finite group and $H$ is a subgroup of $G$, then $|G : H| = |G|/|H|$.

(1) Prove that $A_4$ (with order 12) is a counterexample of the converse Lagrange’s Theorem as follows. Assume there does exist an order 6 subgroup $H \leq A_4$. Take any order 3 element $a \in A_4$ and look at the cosets $H$, $aH$, and $a^2H$. What does the pigeonhole principle say about $H$, $aH$, and $a^2H$? Deduce that in all possible cases that $a \in H$. How many order 3 elements are there? What is the contradiction?

(2) Prove Corollary 2 of Lagrange’s theorem: in a finite group, the order of each element of the group divides the order of the group.

**Corollary 3 of Lagrange’s Theorem**
A group of prime order is cyclic.

**Corollary 4 of Lagrange’s Theorem**
Let $G$ be a finite group, and let $a \in G$. Then $a^{|G|} = e$.

(4) Completely fill out the Cayley table for the group of order 6 which has an element $a$ of order 3, and an element $b$ of order 2 satisfying $ba = a^{-1}b$.
**Theorem 7.2: Classification of Groups of Order $2p$**
Let $G$ be a group of order $2p$, where $p$ is a prime greater than 2. Then $G$ is isomorphic to $Z_{2p}$ or $D_p$.

**Definition: Stabilizer of a Point**
Let $G$ be a group of permutations of a set $S$. For each $i \in S$, let stab$_G(i) = \{ \phi \in G \mid \phi(i) = i \}$. We call stab$_G(i)$ the *stabilizer of $i$ in $G$.*

**Definition: Orbit of a Point**
Let $G$ be a group of permutations of a set $S$. For each $s \in S$, let orb$_G(s) = \{ \phi(s) \mid \phi \in G \}$. The set orb$_G(s)$ is called the *orbit of $s$ under $G$.*

(5) Let $G = A_4$ be the group of even permutations on $\{1, 2, 3, 4\}$. Compute stab$_G(1)$ and orb$_G(1)$. What is the product of the sizes of these two sets?

(6) Let $G$ be $D_4$, the set of plane symmetries of the square with side length 2 centered at the origin. Let $p$ be the point with Cartesian coordinates $(\sqrt{2}/2, \sqrt{2}/2)$. Compute stab$_G(p)$ and orb$_G(p)$. What is the product of the sizes of these two sets?

(7) Let $G$ be a group of permutations of a set $S$ and let $i \in S$. Prove that stab$_G(i)$ is a subgroup of $G$. 