Group Members: ______________________________

Refer to the Cayley Table of D_4 on p. 33, and the table of D_3 below.

(1a) What is the center of D_3? What are the centralizers $C(R_0)$, $C(R_{120})$, and $C(F_1)$?

(1b) What is the center of D_4? What are the centralizers $C(R_{90})$, $C(R_{180})$, and $C(H)$?

D_3	R_0	R_{120}	R_{240}	F_1	F_2	F_3
R_0	R_0	R_{120}	R_{240}	F_1	F_2	F_3
R_{120}	R_{120}	R_{240}	R_0	F_3	F_1	F_2
R_{240}	R_{240}	R_0	R_{120}	F_2	F_3	F_1
F_1	F_1	F_2	F_3	R_0	R_{120}	R_{240}
F_2	F_2	F_3	F_1	R_{240}	R_0	R_{120}
F_3	F_3	F_1	F_2	R_{120}	R_{240}	R_0

(2a) Make a conjecture for the center of the dihedral group D_n for $n \geq 3$ (plane symmetries of the regular n-gon).

(2b) Make a conjecture for the centralizer of a reflection in D_n for $n \geq 3$.

(3) For this question, collect data about orders of elements and about powers of elements of cyclic groups, grouped according to when different powers yield the same element. The groups are $\mathbb{Z}_5$, $\mathbb{Z}_6$, and $\mathbb{Z}_8$.

a	a^1	a^2	a^3	a^4	a^5	$\|a\|$	sets of $\{i, j, k \ldots \mid a^i = a^j = a^k \cdots\}$
0							
1							
2							
3							
4							

a	a^1	a^2	a^3	a^4	a^5	a^6	$\|a\|$	sets of $\{i, j, k \ldots \mid a^i = a^j = a^k \cdots\}$
0								
1								
2								
3								
4								
5								

a	a^1	a^2	a^3	a^4	a^5	a^6	a^7	a^8	$\lvert a\rvert$	sets of $\{i, j, k \ldots \mid a^i = a^j = a^k \cdots\}$
0										
1										
2										
3										
4										
5										
6										
7										

(4a) Conjecture a condition for when $a \in \mathbb{Z}_n$ is a generator of $\mathbb{Z}_n$.

(4a) Conjecture a condition for when $a^i = a^j$ in $\mathbb{Z}_n$.

Break.

Theorem 4.1 Criterion for $a^i = a^j$. Let G be a group, and let $a \in G$. If $|a| = \infty$, then all distinct powers of a are distinct group elements. If $|a| < \infty$, say $|a| = n$, then $\langle a \rangle =$ ________________ and $a^i = a^j$ iff ____________.

Corollary 1 $|a| = |\langle a \rangle|$. For any group element a, $|a| = |\langle a \rangle|$.

Corollary 2 $a^k = e \Rightarrow |a| | k$. Let G be a group and let $a \in G$ be an element of order n. If $a^k = e$, then n divides k.
