PRINT Last name: KEY First name:

Signature: Student ID:

Instructions: Show work for full credit. No notes, calculators, hats, cell phones, HUD lenses, or aids of any kind. 15-minute time limit. By signing your name you agree that all work is your own.

1. Let V be the subset of $\mathbb{R}^4$ (Euclidean 4-space) defined by $V = \{(a, b, c, -b) \ : \ \text{where } a, b, \text{ and } c \text{ are real numbers}\}$. (So V is the set of all vectors from $\mathbb{R}^4$ with the fourth entry equal to the negative of the second entry.) Give the details/calculations for the three steps of the subspace test to show that V is a subspace of $\mathbb{R}$ (Hint: two steps are closure properties).

(1) V is nonempty since $(0,0,0,0) \in V$.

(2) Let $\vec{u} = (a_1, b_1, c_1, -b_1)$ and $\vec{v} = (a_2, b_2, c_2, -b_2) \in V$ then $(a_1+a_2, b_1+b_2, c_1+c_2, -b_1-b_2) = \vec{u}+\vec{v} \in V$ since $-b_1-b_2 =$ negative of b_1+b_2.

(3) Let $\vec{u} = (a, b, c, -b)$ and let k be a scalar. Then $k\vec{u} = (ka, kb, kc, -kb) \in V$ since $-kb =$ negative of kb.

2. The matrix A has reduced row echelon form R, both shown below.
(a) Write down a basis for the row space of A.
(b) Write down a basis for the column space of A.

$$A = \begin{bmatrix} 1 & 2 & 0 & 2 & 5 \\ -2 & -5 & 1 & -1 & -8 \\ 0 & -3 & 3 & 4 & 1 \\ 3 & 6 & 0 & -7 & 2 \end{bmatrix} \qquad R = \left.\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\right\} (a)$$

(a) $\{(1,0,2,0,1), (0,1,-1,0,1), (0,0,0,1,1)\}$

(b) $\{(1,-2,0,3), (2,-5,-3,6), (2,-1,4,-7)\}$

3. Use any method to show whether or not the three vectors $\mathbf{v}_1 = (1, 4, -2)$, $\mathbf{v}_2 = (-2, 3, -1)$, and $\mathbf{v}_3 = (-1, 2, -4)$ are linearly independent.

Set $A = [\vec{v}_1 \ \vec{v}_2 \ \vec{v}_3]$; check $\det(A)$.

$$\begin{vmatrix} 1 & -2 & -1 \\ 4 & 3 & 2 \\ -2 & -1 & -4 \end{vmatrix} = 1(-12+2) - (-2)(-16+4) + (-1)(-4+6)$$
$$= -10 + 2\cdot(-12) - 2 = -36$$

$\det(A) \neq 0 \Rightarrow$ $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ are linearly independent

$$\left(A \xrightarrow[\text{reduce}]{\text{row}} \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & 6/11 \\ 0 & 0 & 1 \end{bmatrix} \right)$$

Math 332-01 | Quiz 2 | Oct. 7, 2010

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Instructions: Show work for full credit. No notes, calculators, hats, cell phones, HUD lenses, or aids of any kind. 15-minute time limit. By signing your name you agree that all work is your own.

1. Let V be the subset of $\mathbb{R}^4$ (Euclidean 4-space) defined by $V = \{(a, b, -a, d) \ : \text{ where } a,\ b, \text{ and } d \text{ are real numbers}\}$. (So V is the set of all vectors from $\mathbb{R}^4$ with the third entry equal to the negative of the first entry.) Give the details/calculations for the three steps of the subspace test to show that V is a subspace of $\mathbb{R}$ (Hint: two steps are closure properties).

(1) V is nonempty since $(0,0,0,0) \in V$.

(2) Let $\vec{u} = (a_1, b_1, -a_1, d_1)$ and $\vec{v} = (a_2, b_2, -a_2, d_2)$.

Then $\vec{u} + \vec{v} = (a_1 + a_2, b_1 + b_2, -a_1 - a_2, d_1 + d_2) \in V$

since $-a_1 - a_2 =$ negative of $a_1 + a_2$.

(3) Let $\vec{u} = (a_1, b_1, -a_1, d_1)$ and let k be a scalar.

Then $k\vec{u} = (ka_1, kb_1, -ka_1, kd_1) \in V$ since

$-ka_1 =$ negative of ka.

2. The matrix A has reduced row echelon form R, both shown below.
(a) Write down a basis for the row space of A.
(b) Write down a basis for the column space of A.

$\vec{r}_3 = -10\vec{r}_1 + 5\vec{r}_2$.

$$A = \begin{bmatrix} 1 & 3 & -2 & 0 & 0 \\ 2 & 6 & -5 & -3 & -2 \\ 0 & 0 & 5 & 15 & 10 \\ 2 & 6 & 6 & 18 & 8 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 3 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ (a)}$$

(b)

(a) $\{(1,3,0,0,-2),$
$(0,0,1,0,-1),$
$(0,0,0,1,1)\}$

(b) $\{(1,2,0,2),$
$(-2,-5,5,6),$
$(0,-3,15,18)\}$

3. Use any method to show whether or not the three vectors $\mathbf{v}_1 = (1, 4, -2)$, $\mathbf{v}_2 = (-2, 3, -1)$, and $\mathbf{v}_3 = (1, 15, -7)$ are linearly independent.

Set $A = [\vec{v}_1\ \vec{v}_2\ \vec{v}_3]$ and check $\det(A) = 0$.

$$\begin{vmatrix} 1 & -2 & 1 \\ 4 & 3 & 15 \\ -2 & -1 & -7 \end{vmatrix} = 1(-21+15) - (-2)(-28+30) + 1(-4+6)$$

$$= -6 + 4 + 2 = 0.$$

$\det(A) = 0 \Rightarrow \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ are linearly dependent.

$$\left(A \xrightarrow[\text{red.}]{\text{row}} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \right)$$