PRINT Last name:_____ First name: KEY

Signature:_____ Student ID:_____

Instructions: Show work for full credit. No notes, calculators, hats, cell phones, trained parrots, or aids of any kind. 15-minute time limit. By signing your name you agree that all work is your own.

1. Is the following matrix invertible? If no, justify your answer with a computation and explain the result. If yes, compute the inverse.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$

$R_3 - 2R1 - R2 \to R3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & -2 & 1 & 1 \end{array}\right]$$

$R3/-1 \to R3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 2 & -1 & -1 \end{array}\right]$$

$R1 - R3 \to R1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 2 & -1 & -1 \end{array}\right]$$

A^{-1}

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R2 \cdot (-1) \to R2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

2. Define A and $\mathbf{b}$ as below. Find a solution to the equation $A\mathbf{x} = \mathbf{b}$. Find a nontrivial solution to the equation $A\mathbf{y} = \mathbf{0}$. Use these two solutions to build a distinct second solution to $A\mathbf{x} = \mathbf{b}$.

$$A = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 8 \end{bmatrix}$$

solve simultaneously

$$\left[\begin{array}{cccc|c|c} 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 4 & 8 & 0 \end{array}\right]$$

$R3/4 \to R3$

$$\left[\begin{array}{cccc|c|c} 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \end{array}\right]$$

$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1+s \\ 1-s \\ s \\ 2 \end{bmatrix}$ $\quad \vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} s \\ -s \\ s \\ 0 \end{bmatrix}$ $\quad (\vec{x}+\vec{y}) = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 2 \end{bmatrix}$

Solves $A\vec{x} = \vec{b}$.

Set $s=0$: $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}$.

Set $s=1$: $\vec{y} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$

3. Compute the following determinant. Show steps!

$$\begin{vmatrix} 0 & 5 & -6 & 7 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 3 & -4 \\ 0 & 0 & -8 & 9 \end{vmatrix}$$

$$= (-1)^3 (1)(5)(-8)(1)$$

$$= \boxed{40}$$

$$\begin{vmatrix} 0 & 5 & -6 & 7 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 3 & -4 \\ 0 & 0 & -8 & 9 \end{vmatrix}$$

$(-1)(-1)$

$$\begin{vmatrix} 1 & -2 & 3 & -4 \\ 0 & 0 & -8 & 9 \\ 0 & 5 & -6 & 7 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$= (-1)^3 \begin{vmatrix} 1 & -2 & 3 & -4 \\ 0 & 5 & -6 & 7 \\ 0 & 0 & -8 & 9 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

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1. Compute the following determinant. Show steps!

$$\begin{vmatrix} 0 & -5 & 6 & -7 \\ -1 & 2 & -3 & 4 \\ 0 & 0 & 8 & -9 \\ 0 & 0 & 0 & -1 \end{vmatrix}$$

$$(-1)\begin{vmatrix} -1 & 2 & -3 & 4 \\ 0 & -5 & 6 & -7 \\ 0 & 0 & 8 & -9 \\ 0 & 0 & 0 & -1 \end{vmatrix} = (-1)(-1)(-5)(8)(-1) = 40$$

2. Define A and $\mathbf{b}$ as below. Find a solution to the equation $A\mathbf{x} = \mathbf{b}$. Find a nontrivial solution to the equation $A\mathbf{y} = \mathbf{0}$. Use these two solutions to build a distinct second solution to $A\mathbf{x} = \mathbf{b}$.

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ -6 \\ 2 \end{bmatrix}$$

solve simultaneously

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 3 & 0 & -6 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \quad \left|\ R_2 \cdot \tfrac{1}{3} \to R_2\right.$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$$\vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} -2s \\ s \\ 0 \\ 0 \end{bmatrix} \qquad s=1 \Rightarrow \vec{y} = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1-2s \\ s \\ -2 \\ 2 \end{bmatrix}$$

$$(\vec{x}+\vec{y}) = \begin{bmatrix} -1 \\ 1 \\ -2 \\ 2 \end{bmatrix}$$ also solves $A\vec{x} = \vec{b}$.

select $s=0$:

$$\vec{x} = \begin{bmatrix} 1 \\ 0 \\ -2 \\ +2 \end{bmatrix}$$

3. Is the following matrix invertible? If no, justify your answer with a computation and explain the result. If yes, compute the inverse.

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 1 & -2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 3 & 1 & -2 & 0 & 0 & 1 \end{array}\right]$$

$R1 \cdot (-1) \to R1$ ↓

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 3 & 1 & -2 & 0 & 0 & 1 \end{array}\right]$$

$R_3 - 3R1 \to R3$

$R_3 - R_2 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 3 & -1 & 1 \end{array}\right]$$

$R1 + R3 \to R1$ ↓

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 3 & -1 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix}$$