

Math 230 Practice for Final

April 29, 2011

DISCLAIMER: See also practice exams 1 & 2. This one contains only material after Exam 2. Also, review all self-assessments and non-challenge suggested problems posted under reading and suggested problems.

In the next three questions below suppose that a “word” is any string of seven letters of the alphabet, with repeated letters allowed.

1. How many words begin with R and end with T?
2. How many words begin with A or B and end with A or B?
3. How many words have exactly one vowel?
4. Find the number of words of length eight of distinct letters of the alphabet so that the words do not have both A and B in them.
5. How many permutations of the seven letters A, B, C, D, E, F, G have E in the first position?
6. How many permutations of the seven letters A, B, C, D, E, F, G do not have the vowels next to each other?
7. A class consists of 20 sophomores and 15 freshmen. The class needs to form a committee of size five.
 - (a) How many committees are possible?
 - (b) How many committees are possible if the committee must have three sophomores and two freshmen?
8. Suppose $A = 4$ and $B = 10$. Find the number of functions $f : A \rightarrow B$.
9. Suppose $A = 4$ and $B = 10$. Find the number of 1-1 functions $f : A \rightarrow B$.
10. Suppose $A = 10$ and $B = 4$. Find the number of 1-1 functions $f : A \rightarrow B$.
11. How many subsets with an odd number of elements does a set with 10 elements have?
12. A professor teaching a Discrete Math course gives a multiple choice quiz that has ten questions, each with four possible responses: a, b, c, d. What is the minimum number of students that must be in the professor’s class in order to guarantee that at least three answer sheets must be identical? (Assume that no answers are left blank.)
13. A computer is programmed to print subsets of $\{1, 2, 3, 4, 5\}$ at random. If the computer prints 40 subsets, prove that some subset must have been printed at least twice.
14. Use the binomial theorem to expand $(x + y)^5$.

15. Use the binomial theorem to prove the following:

$$3^{100} = \binom{100}{0} + \binom{100}{1} \cdot 2 + \binom{100}{2} \cdot 2^2 + \cdots + \binom{100}{99} 2^{99} + \binom{100}{100} \cdot 2^{100}.$$

16. Find the coefficient of x^8 in the expansion of $(x^2 + 2)^{13}$.
17. Find the number of permutations of the word CORRECT.
18. Use the Principle of Mathematical Induction to prove that any integer amount of postage from 18 cents on up can be made from an infinite supply of 4-cent and 7-cent stamps.
19. Use mathematical induction to show that n lines in the plane passing through the same point divide the plane into $2n$ regions.
20. Principle of Mathematical Induction to prove that $4|(9^n - 5^n)$ for all $n \geq 0$.
21. Find the error in the following proof of this “theorem”:
“Theorem: Every positive integer equals the next largest positive integer.”
“Proof: Let $P(n)$ be the proposition ‘ $n = n + 1$ ’. To show that $P(k) \rightarrow P(k + 1)$, assume that $P(k)$ is true for some k , so that $k = k + 1$. Add 1 to both sides of this equation to obtain $k + 1 = k + 2$, which is $P(k + 1)$. Therefore $P(k) \rightarrow P(k + 1)$ is true. Hence $P(n)$ is true for all positive integers n .”

In the questions below give a recursive definition with initial condition(s).

22. The function $f(n) = 2^n$, $n = 1, 2, 3, \dots$
23. The function $f(n) = n!$, $n = 0, 1, 2, \dots$
24. The function $f(n) = 5n + 2$, $n = 1, 2, 3, \dots$
25. The set $\{1, 1/3, 1/9, 1/27 \dots\}$.
26. The set $\{\dots, -4, -2, 0, 2, 4, 6 \dots\}$.
27. Find $f(2)$ and $f(3)$ if $f(n) = 2f(n - 1) + 6$, $f(0) = 3$.
28. Find $f(2)$ and $f(3)$ if $f(n) = f(n - 1) \cdot f(n - 2) + 1$, $f(0) = 1$, $f(1) = 4$.
29. What is the probability that a randomly selected day of a Leap year (366 days) is in May?
30. What is the probability that the sum of the numbers on two dice is even when they are rolled?
31. What is the probability that a fair coin lands Heads 6 times in a row?
32. Find and correct the error in the solution to the following problem:
Problem: You flip two coins and want to find the probability that both coins show heads.
Solution: There are three possible outcomes: 2 heads, 2 tails, or 1 head and 1 tail. Since a “success” is one of these three outcomes, $p(\text{both heads}) = 1/3$.

In the questions below suppose you have 40 different books (20 math books, 15 history books, and 5 geography books).

33. You pick two books at random according to the following procedure: select the first book, put it back, then select the second book. Each book is equally probable to be chosen each time. What is the probability that both books are history books?
34. You pick two books at random according to the same procedure as the previous problem. What is the probability that the two books are from different disciplines?
35. What is the probability that a length 8 bit string will either begin with three 0s or end with two 1s?

In the questions below determine whether the binary relation is: (1) reflexive, (2) symmetric, (3) antisymmetric, (4) transitive.

36. The relation R on $\{1, 2, 3, \dots\}$ where aRb means $a|b$.
37. The relation R on $\{w, x, y, z\}$ where $R = \{(w, w), (w, x), (x, w), (x, x), (x, z), (y, y), (z, y), (z, z)\}$.
38. The relation R on \mathbb{Z} where aRb means $|a - b| \leq 1$.
39. The relation R on \mathbb{Z} where aRb means that the units digit (in decimal representation) of a is equal to the units digit of b .
40. The relation R on the set of all people where aRb means that a is at least as tall as b .

From the definitions of the properties reflexive, symmetric, and transitive for relations, prove whether or not the following relations are equivalence relations; i.e., the following relations are reflexive, symmetric, and transitive. For a disproof, you typically will find a counterexample to one of the properties.

41. $R \subseteq \mathbb{R} \times \mathbb{R}$ defined by xRy provided $|x| = |y|$
42. $R \subseteq \mathbb{R} \times \mathbb{R}$ defined by xRy provided $|x - y| \leq 1$
43. $R \subseteq \mathbb{Z} \times \mathbb{Z}$ defined by aRb provided $a \equiv b \pmod{3}$
44. $R \subseteq \mathbb{R} \times \mathbb{R}$ defined by xRy provided $x \leq y$.
45. (Challenge) $R \subseteq \mathbb{R} \times \mathbb{R}$ defined by xRy provided xy is rational (which properties does this relation have?)