

Euro coins optimal change question

Suppose that you have an unlimited supply of the Euro coins having denominations 50, 20, 10, 5, 2, and 1 cents. If n is a positive integer, then n cents in change composed with the fewest possible number of these coins require that the amount of change in the 20, 10, 5, 2, and 1 cent coins cannot exceed 49 cents.

For you to do: Assume you have change for n cents with the smallest possible number of coins.

(1) Determine and prove an upper bound on the number of individual coins for each of the 20, 10, 5, 2, and 1 cent coins. For example:

Claim: There can be no more than two 20-cent coins;

Proof of claim: Or else you would replace three 20-cent coins with a 50-cent and a 10-cent coins, thereby reducing the total number of coins.

(2) Also, determine and prove any additional restrictions on combinations of more than one coin. (Recall that in the US system, you cannot have both two dimes and one nickel).

DO NOT PROVE the entire Lemma 1 for the Euro coin system.