**Math 230 (Ellis) Spring 2008 Quiz 2** **Name:** Key B

1. What is the smallest integer $n$ such that $f(x)$ is $O(x^n)$ where $f(x) = (x+1)(x+2)x + x^2(x^2+5)$?

   (a) 6
   (b) 4
   (c) 3
   (d) 1

$(x+1)(x+2)x + x^2(x^2+5)$

$x^3 + x^4$

$x^4$

2. Find witnesses $C$ and $k$ that demonstrate that $f(x) = 3x^2 + 5x$ is $O(x^2)$. (Hint: $x > k \rightarrow |f(x)| \leq C|x^2|$.)

Let $k=1$. Then $3x^2+5x \leq 3x^2+5x^2 = 8x^2$, and everything is positive.

so let $C=8$.

$(\forall x > 1,\ |3x^2+5x| \leq 3|x^2|+5|x| = 3x^2+5x \leq 8x^2 = 8|x^2|)$

3. Which of these statements is true?

   (a) If $f$ is $O(g)$, then $g$ is $O(f)$.
   (b) If $f$ is $O(g)$, then $g$ is not $O(f)$.
   (c) If $f$ is $O(g)$, then $f$ is $O(g/2)$.
   (d) If $f_1$ and $f_2$ are $O(g)$, then $f_1 f_2$ is $O(g)$.

4. The bubble sort algorithm is as follows.

**procedure** *bubblesort*($a_1, \ldots, a_n$: real numbers with $n \geq 2$)
**for** $i := 1$ **to** $n-1$
  **for** $j := 1$ **to** $n-1$
    **if** $a_j > a_{j+1}$ **then** interchange $a_j$ and $a_{j+1}$
{$a_1, \ldots, a_n$ is in increasing order }

If the input list is $4, 2, 1, 3$, give the list after the for loop completes each value of $i$.

before loop: 4 2 1 3

after $i=1$: 2 4 1 3 → 2 1 4 3 → 2 1 3 4

after $i=2$: 1 2 3 4

after $i=3$: 1 2 3 4

5. A cashier using a strange currency system has coins worth 7, 5, and 1 cents. Give a value for which the greedy way of making change gives a larger number of coins than the optimal way of making change. Write down both ways of making change, labeling one "greedy" and the other "optimal".

10¢ = 7¢ + 1¢ + 1¢ + 1¢ (greedy)
= 5¢ + 5¢ (optimal)

11¢ = 7¢ + 4·1¢ (g)
-OR- = 2·5¢ + 1¢ (o)

6. For the bubble sort algorithm (written elsewhere in this quiz), write down the average case time complexity and the worst case time complexity using the notation $\Theta(f(n))$ for the appropriate function $f(n)$. (There will be two answers, clearly label them "average case" and "worst case".)

average case = $\Theta(n^2)$
worst case = $\Theta(n^2)$

(when counting comparisons)
see other key for explanation.

**PMA (Public Mathematics Announcement)**

12:50-1:40pm - Math Club with Professor Trefethen, Life Sciences Building, Room 111 (lunch provided!)
3:15-5:30pm Menger Day talks in McCloska Ballroom, MTCC