

1. What is the smallest integer n such that $f(x)$ is $O(x^n)$ where $f(x) = (x+1)(x+2)x + x^2(x^2+5)$?

- (a) 6
- (b) 4
- (c) 3
- (d) 1

$$\begin{aligned} & (x+1)(x+2)x + x^2(x^2+5) \\ & \quad \swarrow \quad \searrow \quad \downarrow \quad \quad \downarrow \\ & \quad \quad \quad x^3 \quad + \quad x^4 \\ & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \downarrow \\ & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad x^4 \end{aligned}$$

2. Find witnesses C and k that demonstrate that $f(x) = 3x^2 + 5x$ is $O(x^2)$. (Hint: $x > k \rightarrow |f(x)| \leq C|x^2|$.)

Let $k=1$. Then $3x^2 + 5x \leq 3x^2 + 5x^2 = 8x^2$,
and everything is positive.

so let $C=8$.

$$(\forall x > 1, |3x^2 + 5x| \leq 3|x^2| + 5|x| = 3x^2 + 5x \leq 8x^2 = 8|x^2|)$$

3. Which of these statements is true?

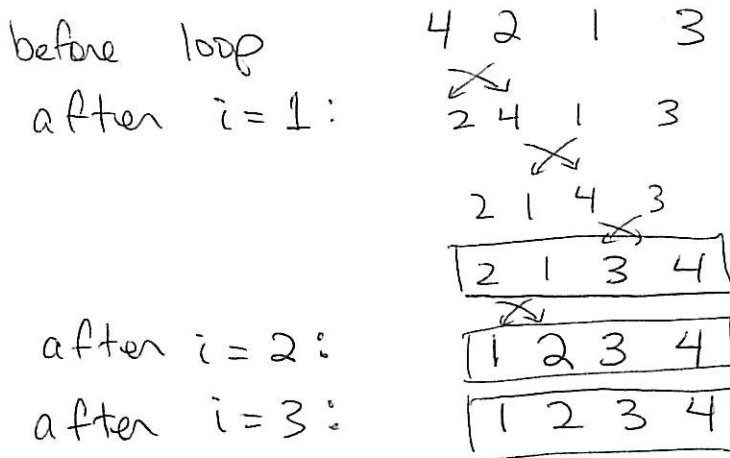
- (a) If f is $O(g)$, then g is $O(f)$.
- (b) If f is $O(g)$, then g is not $O(f)$.
- (c) If f is $O(g)$, then f is $O(g/2)$.
- (d) If f_1 and f_2 are $O(g)$, then $f_1 f_2$ is $O(g)$.

4. The bubble sort algorithm is as follows.

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procedure bubblesort( $a_1, \dots, a_n$ : real numbers with  $n \geq 2$ )
for  $i := 1$  to  $n - 1$ 
  for  $j := 1$  to  $n - 1$ 
    if  $a_j > a_{j+1}$  then interchange  $a_j$  and  $a_{j+1}$ 
  { $a_1, \dots, a_n$  is in increasing order }
    
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If the input list is 4, 2, 1, 3, give the list after the for loop completes each value of i .



5. A cashier using a strange currency system has coins worth 7, 5, and 1 cents. Give a value for which the greedy way of making change gives a larger number of coins than the optimal way of making change. Write down both ways of making change, labeling one "greedy" and the other "optimal".

$$10¢ = 7¢ + 1¢ + 1¢ + 1¢ \quad (\text{greedy})$$

$$= 5¢ + 5¢ \quad (\text{optimal})$$

$$11¢ = 7¢ + 4 \cdot 1¢ \quad (g)$$

$$= 2 \cdot 5¢ + 1¢ \quad (o)$$

6. For the bubble sort algorithm (written elsewhere in this quiz), write down the average case time complexity and the worst case time complexity using the notation $\Theta(f(n))$ for the appropriate function $f(n)$. (There will be two answers, clearly label them "average case" and "worst case".)

$$\text{average case} = \Theta(n^2)$$

$$\text{worst case} = \Theta(n^2)$$

(when counting comparisons)

see other key for explanation.

PMA (Public Mathematics Announcement)

12:50-1:40pm - Math Club with Professor Trefethen, Life Sciences Building, Room 111 (lunch provided!)

3:15-5:30pm Menger Day talks in McCloska Ballroom, MTCC