Math 230 (Ellis) Spring 2008 Quiz 2. Name: Key A

1. What is the smallest integer n such that $f(x)$ is $O(x^n)$ where $f(x) = (x+1)(x^3+2)x + x^2(x^2+5)$?

   (a) 2
   (b) 4
   [✓ (c) 5]
   (d) 7

$$(x+1)(x^3+2)x + x^2(x^2+5)$$
$$x^5 + x^4$$
$$x^5$$

2. Find witnesses C and k that demonstrate that $f(x) = 5x^2 + 3x$ is $O(x^2)$. (Hint: $x > k \rightarrow |f(x)| \leq C|x^2|$.)

Let $\boxed{k=1.}$ Then $5x^2+3x \leq 5x^2+3x^2 = 8x^2$
and everything is positive

So let $\boxed{C=8.}$

$(\forall x>1, \quad |5x^2+3x| \leq 5|x^2|+3|x| = 5x^2+3x \leq 8x^2 = 8|x^2|)$

3. Which of these statements is true?

   (a) If f_1 and f_2 are $O(g)$, then $f_1 f_2$ is $O(g)$.
   [✓ (b) If f is $O(g)$, then f is $O(g/2)$.]
   (c) If f is $O(g)$, then g is $O(f)$.
   (d) If f is $O(g)$, then g is not $O(f)$.

4. The bubble sort algorithm is as follows.

note: book gives j:=1 to n-i

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procedure bubblesort($a_1, \ldots, a_n$: real numbers with $n \geq 2$)
for $i := 1$ to $n - 1$
    for $j := 1$ to $n - 1$
        if $a_j > a_{j+1}$ then interchange $a_j$ and $a_{j+1}$
{$a_1, \ldots, a_n$ is in increasing order }
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If the input list is $3, 2, 4, 1$, give the list after the for loop completes each value of i.

Before loop	3	2	4	1
after $i=1$	2	3	1	4
after $i=2$	2	1	3	4
after $i=3$	1	2	3	4

5. A cashier using a strange currency system has coins worth 9, 7, and 1 cents. Give a value for which the greedy way of making change gives a larger number of coins than the optimal way of making change. Write down both ways of making change, labeling one "greedy" and the other "optimal".

Answer 1

14¢ = 9¢ + 1¢ + 1¢ + 1¢ + 1¢ + 1¢ (greedy)

= 7¢ + 7¢ (optimal)

Other Answers

15¢ = 9¢ + 6·1¢ (g)
= 2·7¢ + 1¢ (o)

22¢ = 2·9¢ + 4·1¢ (g)
= 3·7¢ + 1¢ (o)

6. For the bubble sort algorithm (written elsewhere in this quiz), write down the average case time complexity and the worst case time complexity using the notation $\Theta(f(n))$ for the appropriate function $f(n)$. (There will be two answers, clearly label them "average case" and "worst case".)

average case = $\Theta(n^2)$
worst case = $\theta(n^2)$

(when counting comparisons)
explanation not required for credit.

Fixed i: $a_j > a_{j+1}$ compared $n-1$ times. j compared to loop maximum $n-1$ times when $j = 1, 2, \ldots, n-1$, and one final time for exit. Total: $2n-1$ times.

Total: $(n-1)\cdot(2n-1)$. Then i is compared to loop maximum $n-1$ times (when $i = 1, 2, \ldots, n-1$) and one final time for exit. Total: $(n-1)(2n-1) + n$ $\boxed{= \Theta(n^2)}$

PMA (Public Mathematics Announcement)

12:50-1:40pm - Math Club with Professor Trefethen, Life Sciences Building, Room 111 (lunch provided!)
3:15-5:30pm Menger Day talks in McCloska Ballroom, MTCC