

1. What is the smallest integer n such that $f(x)$ is $O(x^n)$ where $f(x) = (x+1)(x^3+2)x + x^2(x^2+5)$?

- (a) 2
- (b) 4
- (c) 5
- (d) 7

$$(x+1)(x^3+2)x + x^2(x^2+5)$$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $x^5 \quad + \quad x^4$
 \downarrow
 x^5

2. Find witnesses C and k that demonstrate that $f(x) = 5x^2 + 3x$ is $O(x^2)$. (Hint: $x > k \rightarrow |f(x)| \leq C|x^2|$.)

Let $k=1$. Then $5x^2 + 3x \leq 5x^2 + 3x^2 = 8x^2$
and every thing is positive

So let $C=8$.

$$(\forall x > 1, |5x^2 + 3x| \leq 5|x^2| + 3|x| = 5x^2 + 3x \leq 8x^2 = 8|x^2|)$$

3. Which of these statements is true?

- (a) If f_1 and f_2 are $O(g)$, then $f_1 f_2$ is $O(g)$.
- (b) If f is $O(g)$, then f is $O(g/2)$.
- (c) If f is $O(g)$, then g is $O(f)$.
- (d) If f is $O(g)$, then g is not $O(f)$.

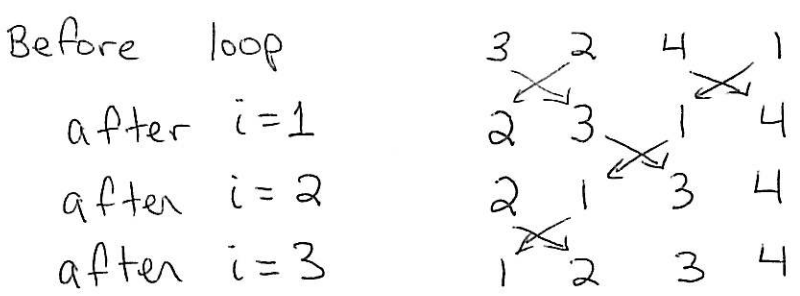
4. The bubble sort algorithm is as follows.

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procedure bubblesort( $a_1, \dots, a_n$ : real numbers with  $n \geq 2$ )
for  $i := 1$  to  $n - 1$ 
  for  $j := 1$  to  $n - 1$ 
    if  $a_j > a_{j+1}$  then interchange  $a_j$  and  $a_{j+1}$ 
  { $a_1, \dots, a_n$  is in increasing order }
    
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note: book gives $j := 1$ to $n-i$

If the input list is 3, 2, 4, 1, give the list after the for loop completes each value of i .



5. A cashier using a strange currency system has coins worth 9, 7, and 1 cents. Give a value for which the greedy way of making change gives a larger number of coins than the optimal way of making change. Write down both ways of making change, labeling one "greedy" and the other "optimal".

Answer 1

$$14¢ = 9¢ + 1¢ + 1¢ + 1¢ + 1¢ + 1¢ \quad (\text{greedy})$$

$$= 7¢ + 7¢ \quad (\text{optimal})$$

Other Answers

$$15¢ = 9¢ + 6 \cdot 1¢ \quad (g)$$

$$= 2 \cdot 7¢ + 1¢ \quad (o)$$

$$22¢ = 2 \cdot 9¢ + 4 \cdot 1¢ \quad (g)$$

$$= 3 \cdot 7¢ + 1¢ \quad (o)$$

6. For the bubble sort algorithm (written elsewhere in this quiz), write down the average case time complexity and the worst case time complexity using the notation $\Theta(f(n))$ for the appropriate function $f(n)$. (There will be two answers, clearly label them "average case" and "worst case".)

average case = $\Theta(n^2)$

worst case = $\Theta(n^2)$

(when counting comparisons)
 explanation, not required for credit.

Fixed i : $a_j > a_{j+1}$ compared $n-1$ times, j compared to loop maximum $n-1$ times when $j=1, 2, \dots, n-1$, and one final time for exit. Total: $2n-1$ times.

Total: $(n-1) \cdot (2n-1)$. Then i is compared to loop maximum $n-1$ times (when $i=1, 2, \dots, n-1$) and one final time

PMA (Public Mathematics Announcement) for exit. Total: $(n-1)(2n-1) + n$

$\Theta(n^2)$