

1. What is the smallest integer  $n$  such that  $f(x)$  is  $O(x^n)$  where  $f(x) = (x + 1)(x^3 + 2)x + x^2(x^2 + 5)$ ?
  - (a) 2
  - (b) 4
  - (c) 5
  - (d) 7
  
2. Find witnesses  $C$  and  $k$  that demonstrate that  $f(x) = 5x^2 + 3x$  is  $O(x^2)$ . (Hint:  $x > k \rightarrow |f(x)| \leq C|x^2|$ .)
  
  
  
  
  
  
  
  
  
  
3. Which of these statements is true?
  - (a) If  $f_1$  and  $f_2$  are  $O(g)$ , then  $f_1 f_2$  is  $O(g)$ .
  - (b) If  $f$  is  $O(g)$ , then  $f$  is  $O(g/2)$ .
  - (c) If  $f$  is  $O(g)$ , then  $g$  is  $O(f)$ .
  - (d) If  $f$  is  $O(g)$ , then  $g$  is not  $O(f)$ .

4. The bubble sort algorithm is as follows.

```
procedure bubblesort( $a_1, \dots, a_n$ : real numbers with  $n \geq 2$ )  
for  $i := 1$  to  $n - 1$   
  for  $j := 1$  to  $n - 1$   
    if  $a_j > a_{j+1}$  then interchange  $a_j$  and  $a_{j+1}$   
  { $a_1, \dots, a_n$  is in increasing order }
```

If the input list is 3, 2, 4, 1, give the list after the for loop completes each value of  $i$ .

5. A cashier using a strange currency system has coins worth 9, 7, and 1 cents. Give a value for which the greedy way of making change gives a larger number of coins than the optimal way of making change. Write down both ways of making change, labeling one “greedy” and the other “optimal”.
6. For the bubble sort algorithm (written elsewhere in this quiz), write down the average case time complexity and the worst case time complexity using the notation  $\Theta(f(n))$  for the appropriate function  $f(n)$ . (There will be two answers, clearly label them “average case” and “worst case”.)

**PMA (Public Mathematics Announcement)**

12:50-1:40pm - Math Club with Professor Trefethen, Life Sciences Building, Room 111 (lunch provided!)

3:15-5:30pm Menger Day talks in McCloska Ballroom, MTCC