Math 230 (Ellis) Spring 2008 Quiz 1, Chapter 1. **Name:**____________________

1. Write out the negation of the following proposition, so that the negation symbol ('¬') appears only in front of a propositional variable and never in front of a parenthesis.

   $(p \wedge q) \rightarrow (q \leftrightarrow \neg s)$

2. Suppose $P(x, y)$ is a predicate where the universe for x and y is $\{1, 2, 3\}$. Assume that the predicate is true exactly in the cases – $P(1, 3)$, $P(3, 2)$, $P(2, 1)$, $P(1, 2)$, $P(2, 2)$ – and false otherwise. Determine which of the following quantified statements is FALSE.

   (a) $\exists x \forall y P(x, y)$
   (b) $\forall x \exists y P(x, y)$
   (c) $\exists y \forall x P(x, y)$
   (d) $\forall y \exists x P(x, y)$

3. Suppose you wish to prove the following about integers x and y.

   If x is even and y is odd, then $x^2 - 3xy + 1$ is odd

   Circle the most accurate statement.

   (a) The statement can be proved easily by direct proof but with difficulty by contrapositive.
   (b) The statement can be proved easily by contrapositive but with difficulty by direct proof.
   (c) The statement can be proved easily by either direct proof or contrapositive.
   (d) Both direct proof and contrapositive will be difficult.

4. Suppose you are examining a statement of the form $\forall x(P(x) \rightarrow Q(x))$. If you are looking for a counter example, you need to find a value of x such that

   (a) $P(x)$ is true and $Q(x)$ is true
   (b) $P(x)$ is true and $Q(x)$ is false
   (c) $P(x)$ is false and $Q(x)$ is true
   (d) $P(x)$ is false and $Q(x)$ is false

5. Circle the letter of the statement which says that

   Every nonzero real number has a unique multiplicative inverse.

   Assume the universe for quantification consists of all real numbers.

   (a) $\forall x \exists y (xy = 1)$
   (b) $\forall x \exists y \exists z[(xy = 1) \wedge (xz = 1)]$
   (c) $\forall x \exists y \forall z[((x \neq 0) \wedge (xy = 1)) \wedge ((xz = 1) \rightarrow (y = z))]$
   (d) $\forall x \forall y \exists z[(xy = yz) \rightarrow (x \neq 0)]$

6. Write down two distinct propositions which are logically equivalent to $\neg p \to q$.

7. Circle the letter corresponding to the negation of

$$\exists x \forall y \exists z \big(P(x,y) \oplus Q(x,z)\big)$$

(a) $\forall x \exists y \forall z \big(P(x,y) \oplus Q(x,z)\big)$
(b) $\exists x \forall y \exists z \big(P(x,y) \leftrightarrow Q(x,z)\big)$
(c) $\forall x \exists y \forall z \big(P(x,y) \leftrightarrow Q(x,z)\big)$
(d) $\exists x \forall y \exists z \big(\neg P(x,y) \oplus \neg Q(x,z)\big)$
(e) $\forall x \exists y \forall z \big(\neg P(x,y) \oplus \neg Q(x,z)\big)$

8. Suppose you wish to prove the following about integers x and y by contraposition (contrapositive).

If x is odd and y is odd, then $x - y$ is even.

Circle the letter of the hypothesis that you will begin with.

(a) x is odd and y is odd
(b) $x - y$ is odd
(c) $x - y$ is even
(d) x is even or y is even
(e) x is odd, y is odd, and $x - y$ is odd

9. Let p be the proposition "The train is late", and let q be the proposition "The boss is on time", and let r be the proposition "Alex gets in before the boss". Circle the letter of the expression which is logically equivalent the following proposition in symbols:

If the train is on time or the boss is late then Alex gets in before the boss.

(a) $(p \wedge q) \to r$
(b) $(p \wedge q) \vee r$
(c) $r \to (\neg p \vee \neg q)$
(d) $\neg(p \wedge q) \to r$
(e) $(\neg p \vee \neg q) \vee \neg q$

10. Circle the letter of the negation of the following statement:

There exists a unique star in our solar system.

(a) There are two stars in our solar system.
(b) Either there is no star in our solar system, or there are at least two.
(c) You must consider every star in our solar system as equivalent.
(d) There are many stars in our solar system, and at least two of them are distinct.
(e) There is no star in our solar system.

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(d) $\forall x \exists y (xy = 1)$