

1. Write out the negation of the following proposition, so that the negation symbol (' \neg ') appears only in front of a propositional variable and never in front of a parenthesis.

$(p \wedge q) \rightarrow (q \leftrightarrow \neg s)$ Note: $\neg(p \rightarrow q) \equiv p \wedge \neg q$

$\neg((p \wedge q) \rightarrow (q \leftrightarrow \neg s)) \equiv (p \wedge q) \wedge \neg(q \leftrightarrow \neg s) \equiv \boxed{(p \wedge q) \wedge (q \leftrightarrow s)}$

a 2. Suppose $P(x, y)$ is a predicate where the universe for x and y is $\{1, 2, 3\}$. Assume that the predicate is true exactly in the cases - $P(1, 3), P(3, 2), P(2, 1), P(1, 2), P(2, 2)$ - and false otherwise. Determine which of the following quantified statements is FALSE.

- (a) $\exists x \forall y P(x, y)$
- (b) $\forall x \exists y P(x, y)$
- (c) $\exists y \forall x P(x, y)$
- (d) $\forall y \exists x P(x, y)$

a 3. Suppose you wish to prove the following about integers x and y .

If x is even and y is odd, then $x^2 - 3xy + 1$ is odd

Circle the most accurate statement.

- (a) The statement can be proved easily by direct proof but with difficulty by contrapositive.
- (b) The statement can be proved easily by contrapositive but with difficulty by direct proof.
- (c) The statement can be proved easily by either direct proof or contrapositive.
- (d) Both direct proof and contrapositive will be difficult.

b 4. Suppose you are examining a statement of the form $\forall x(P(x) \rightarrow Q(x))$. If you are looking for a counter example, you need to find a value of x such that

- (a) $P(x)$ is true and $Q(x)$ is true
- (b) $P(x)$ is true and $Q(x)$ is false
- (c) $P(x)$ is false and $Q(x)$ is true
- (d) $P(x)$ is false and $Q(x)$ is false

5. Circle the letter of the statement which says that

Every nonzero real number has a unique multiplicative inverse.

no correct answer is listed.

Assume the universe for quantification consists of all real numbers.

- (a) $\forall x \exists y(xy = 1)$
- (b) $\forall x \exists y \exists z[(xy = 1) \wedge (xz = 1)]$
- (c) $\forall x \exists y \forall z[((x \neq 0) \wedge (xy = 1)) \wedge ((xz = 1) \rightarrow (y = z))]$
- (d) $\forall x \forall y \exists z[(xy = yz) \rightarrow (x \neq 0)]$

$\forall x \exists y \forall z (x \neq 0 \rightarrow [xy = 1 \wedge (xz = 1 \rightarrow y = z)])$

Free point

6. Write down two distinct propositions which are logically equivalent to $\neg p \rightarrow q$.

$$p \vee q$$
$$\neg q \rightarrow p$$

c 7. Circle the letter corresponding to the negation of

$$\exists x \forall y \exists z (P(x, y) \oplus Q(x, z))$$

- (a) $\forall x \exists y \forall z (P(x, y) \oplus Q(x, z))$
- (b) $\exists x \forall y \exists z (P(x, y) \leftrightarrow Q(x, z))$
- (c) $\forall x \exists y \forall z (P(x, y) \leftrightarrow Q(x, z))$
- (d) $\exists x \forall y \exists z (\neg P(x, y) \oplus \neg Q(x, z))$
- (e) $\forall x \exists y \forall z (\neg P(x, y) \oplus \neg Q(x, z))$

b 8. Suppose you wish to prove the following about integers x and y by contraposition (contrapositive).

If x is odd and y is odd, then $x - y$ is even.

Circle the letter of the hypothesis that you will begin with.

- (a) x is odd and y is odd
- (b) $x - y$ is odd
- (c) $x - y$ is even
- (d) x is even or y is even
- (e) x is odd, y is odd, and $x - y$ is odd

d 9. Let p be the proposition "The train is late", and let q be the proposition "The boss is on time", and let r be the proposition "Alex gets in before the boss". Circle the letter of the expression which is logically equivalent the following proposition in symbols:

If the train is on time or the boss is late then Alex gets in before the boss.

- (a) $(p \wedge q) \rightarrow r$
- (b) $(p \wedge q) \vee r$
- (c) $r \rightarrow (\neg p \vee \neg q)$
- (d) $\neg(p \wedge q) \rightarrow r$
- (e) $(\neg p \vee \neg q) \vee \neg r$

b 10. Circle the letter of the negation of the following statement:

There exists a unique star in our solar system.

- (a) There are two stars in our solar system.
- (b) Either there is no star in our solar system, or there are at least two.
- (c) You must consider every star in our solar system as equivalent.
- (d) There are many stars in our solar system, and at least two of them are distinct.
- (e) There is no star in our solar system.