

# Math 230 Practice Exam 1\*

March 11, 2008

**These problems are fairly routine.**

1. What is the truth value of  $(p \vee q) \rightarrow (p \wedge q)$  when both  $p$  and  $q$  are false?
2. What are the converse and contrapositive of the statement “If it is sunny, then I will go swimming”?
3. Show that  $\neg(p \vee \neg q)$  and  $q \wedge \neg p$  are equivalent
  - (a) using a truth table.
  - (b) using logical equivalences.
4. Suppose that  $Q(x)$  is the statement “ $x + 1 = 2x$ .” What are the truth values of  $\forall x Q(x)$  and  $\exists x Q(x)$ ?
5. Prove each of the following statements.
  - (a) The sum of two even integers is always even.
  - (b) The sum of an even integer and an odd integer is always odd.
6. Prove that there are no solutions in positive integers to the equation  $x^4 + y^4 = 100$ .
7. Let  $A = \{a, c, e, h, k\}$ ,  $B = \{a, b, d, e, h, i, k, l\}$ , and  $C = \{a, c, e, i, m\}$ . Find each of the following sets.
  - (a)  $A \cap B$
  - (b)  $A \cap B \cap C$
  - (c)  $A \cup C$
  - (d)  $A \cup B \cup C$
  - (e)  $A - B$
  - (f)  $A - (B - C)$
8. Prove or disprove that if  $A$ ,  $B$ , and  $C$  are sets then  $A - (B \cap C) = (A - B) \cap (A - C)$ .
  - (a) Prove by set containment in both directions, and
  - (b) Prove by set membership table.
9. Let  $f(n) = 2n + 1$ . Is  $f$  a one-to-one function from the set of integers to the set of integers? Is  $f$  an onto function from the set of integers to the set of integers? Explain the reasons behind your answers.
10. Suppose that  $f$  is the function from the set  $\{a, b, c, d\}$  to itself with  $f(a) = d$ ,  $f(b) = a$ ,  $f(c) = b$ ,  $f(d) = c$ . Find the inverse of  $f$ .

**These problems are a bit more challenging.**

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Chapter 1 Supplementary Exercises, pp106-108: 12, 16, 32

Section 2.3: 29ab

Chapter 2 Supplementary Exercises, pp164-165: 14, 18