I. Short answer (3pts each). Do not give proofs.

1. In the best big-oh notation, what is the number of comparisons needed in an iterative algorithm that computes the maximum element of a list of n integers? $O(n)$

2. In the best big-oh notation, what is the number of comparisons needed in the binary search algorithm which searches for a given x in a sorted list of n numbers? $O(\log_2 n)$

3. Name an algorithm that we studied for which the best-case time complexity was different from the worst-case time complexity. linear search

4. **YES** **NO** (Circle one). Is 137 prime?

try $2, 3, 5, 7, 11 = \lfloor \sqrt{137} \rfloor$

5. For the arithmetic progression $5e/3$, $7e/3$, $9e/3$, $11e/3$, $\ldots$, give the *common difference*: $2e/3$

$\frac{7e}{3} - \frac{5e}{3} = \frac{2e}{3}$

6. For the geometric progression $-4, 1, -1/4, 1/16, \ldots$, give the *initial term*: -4 and the *common ratio*: $-1/4$

$r = \frac{1}{-4}$

7. Compute: $\gcd(0, 8) =$ 8 $\gcd(3^6, 3^3) =$ 3^3 $\gcd(20, 50) =$ 10

$\gcd(2^2 \cdot 5, 2 \cdot 5^2)$

8. Compute: $\operatorname{lcm}(20, 50) =$ 100 $\operatorname{lcm}(7, 17) =$ 119 $\operatorname{lcm}(3^4 7^2, 7^4 3^2) =$ $3^4 7^4$

$\operatorname{lcm}(2^2 \cdot 5, 2 \cdot 5^2)$

9. TRUE FALSE (Circle one). In the US coin system, which consists of 1, 5, 10, 25, and 50 cent coins, the greedy way of making change is also the optimal way (when plenty of all coins are available).

10. TRUE FALSE (Circle one). Insertion sort uses $O(n^3)$ comparisons to sort a list of length n.

$O(n^2)$

11. Compute: $10110_2 + 1010_2 =$ 100000_2 $A6_{16} + 4C_{16} =$ $F2_{16}$

$$\begin{array}{r} 10110 \\ 1010 \\ \hline 100000 \end{array} \qquad \begin{array}{r} {}^{1}A6 \\ 4C \\ \hline F2 \end{array}$$

12. Compute: $24 \bmod 11 =$ 2 $-18 \bmod 11 =$ 4

$24 = 2 \cdot 11 + 2$ $-18 = -2 \cdot 11 + 4$

II. Computation Problems 13-16 (10pts each). Show work to clearly justify your answer.

13. Let $f(n) = 5n^2 + 3n + 6$. Give witnesses C and k that show $f(n)$ is $O(n^2)$. (Hint: $n > k \to |f(n)| \leq C|n^2|$.)

when $\boxed{k=1}$, $n > k$

$$\begin{array}{rcl} 5n^2 & \leq & 5n^2 \\ 3n & \leq & 3n^2 \\ 6 & \leq & 6n^2 \\ \hline & & 14n^2 \end{array}$$

$\boxed{C=14}$

$$\left(|5n^2+3n+6| \leq |5n^2| + |3n| + |6| \underset{\substack{\uparrow \\ n>1=k}}{<} 5n^2+3n^2+6n^2 = 14n^2 = C|n^2| \right)$$

14. Find $\sum_{j=1}^{20} 4j$ and $\sum_{i=2}^{20} 3^i$.

$\sum_{j=1}^{20} 4j = 4\sum_{j=1}^{20} j = 4\,\frac{20(21)}{2} = 2\cdot 20\cdot 21 = \boxed{840}$

$\sum_{i=2}^{20} 3^i = \boxed{\frac{3^2 - 3^{21}}{1-3}} \qquad \left(\frac{a_2 - a_{21}}{1-r}\right)$

15. Trace through the Bubble Sort algorithm on the list 2, 4, 1, 3 by writing down the order of the list after each increment of i or j.

procedure bubblesort($a_1, \ldots, a_n$: real with $n \geq 2$)
for $i := 1$ **to** $n - 1$
  for $j := 1$ **to** $n - i$
    if $a_j > a_{j+1}$ **then** interchange a_j and a_{j+1}

Trace

initial list:	2	4	1	3
$i = 1, j = 1$:	2	4	1	3
$i = 1, j = 2$:	2	1	4	3
$i=1, j=3$:	2	1	3	4
$i=2, j=1$:	1	2	3	4
$i=2, j=2$:	1	2	3	4
$i=3, i=1$:	1	2	3	4

16. Use the Euclidean algorithm to compute gcd(120, 78).

$120 = 1\cdot 78 + 42$
$78 = 1\cdot 42 + 36$
$42 = 1\cdot 36 + \boxed{6}$
$36 = 6\cdot 6 + 0$

$\boxed{\gcd(120, 78) = 6}$

III. Proofs (12pts each). Partial credit for good proof structure.

17. Prove that $2^n < n!$ for all integers $n \geq 4$.

See other Key

18. (Recall that by definition, if a and b are integers and m is a positive integer, then $a \equiv b \pmod{m}$ provided that m divides $a - b$.) Prove the following. Let a, b, c be integers, and let m be a positive integer. Show that if $a \equiv b \pmod{m}$, then $a - c \equiv b - c \pmod{m}$.