PRINT Last name:____________________ First name:____________________

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Math 230 Exam 2, Spring 2008

Don't forget to complete online course evaluations at *survey.iit.edu* – April 24-May 11.

I. Short answer (3pts each). Do not give proofs.

1. **TRUE** **FALSE** (Circle one). In the US coin system, which consists of 1, 5, 10, 25, and 50 cent coins, the greedy way of making change is also the optimal way (when plenty of all coins are available).

2. **TRUE** **FALSE** (Circle one). Insertion sort uses $O(n^3)$ comparisons to sort a list of length n.

3. **YES** **NO** (Circle one). Is 131 prime?

4. In the best big-oh notation, what is the number of comparisons needed in an iterative algorithm that computes the maximum element of a list of n integers? __________

5. In the best big-oh notation, what is the number of comparisons needed in the binary search algorithm which searches for a given x in a sorted list of n numbers? __________

6. Name an algorithm that we studied for which the best-case time complexity was different from the worst-case time complexity. ____________________________

7. Compute: $\gcd(12, 18) =$__________ $\gcd(2^5, 2^8) =$__________ $\gcd(5, 0) =$__________

8. Compute: $\mathrm{lcm}(11, 13) =$__________ $\mathrm{lcm}(2^2 5^5, 2^5 5^2) =$__________ $\mathrm{lcm}(12, 18) =$__________

9. For the arithmetic progression $3\pi/4,\ 5\pi/4,\ 7\pi/4,\ 9\pi/4,\ \ldots$, give the
common difference: _________

10. For the geometric progression $2, -1/2, 1/8, -1/32, \ldots$, give the
initial term: _________ and the
common ratio: ________

11. Compute: $-9 \bmod 7 =$__________ $25 \bmod 7 =$__________

12. Compute: $A9_{16} + 3C_{16} =$__________ $10010_2 + 1011_2 =$__________

II. Computation Problems 13-16 (10pts each). Show work to clearly justify your answer.

13. Use the Euclidean algorithm to compute $\gcd(116, 76)$.

14. Trace through the Bubble Sort algorithm on the list $3, 1, 4, 2$ by writing down the order of the list after each increment of i or j.

procedure bubblesort($a_1, \ldots, a_n$: real with $n \geq 2$)
for $i := 1$ **to** $n - 1$
  for $j := 1$ **to** $n - i$
    if $a_j > a_{j+1}$ **then** interchange a_j and a_{j+1}

<u>Trace</u>

initial list:	3	1	4	2
$i = 1, j = 1$:				
$i = 1, j = 2$:				
$\vdots$				

15. Find $\displaystyle\sum_{j=1}^{50} 2j$ and $\displaystyle\sum_{i=2}^{10} 4^i$.

16. Let $f(n) = 3n^2 + 8n + 10$. Give witnesses C and k that show $f(n)$ is $O(n^2)$. (Hint: $n > k \to |f(n)| \leq C|n^2|$.)

III. Proofs (12pts each). Partial credit for good proof structure.

17. (Recall that by definition, if a and b are integers and m is a positive integer, then $a \equiv b \pmod{m}$ provided that m divides $a-b$.) Prove the following. Let a, b, c be integers, and let m be a positive integer. Show that if $a \equiv b \pmod{m}$, then $a - c \equiv b - c \pmod{m}$.

18. Prove that $2^n < n!$ for all integers $n \geq 4$.