PRINT Last name:________________ First name:________________

Signature:________________ Student ID:________________

Math 152 Exam 4, Fall 2007

Instructions. You must show work in order to receive full credit. Partial credit is possible for work which makes positive progress toward the solution.

Conditions. ***No calculators***, notes, books, or scratch paper. By writing your name on the exam you certify that all work is your own, under penalty of all remedies outlined in the student handbook. Please do not talk until after leaving the room.

Time limit: 1 hour 15 minutes (strict).

NOTE: The topics may not be in order either of increasing difficulty or of the order they were covered in the course. Problems are **10 pts each** except where indicated.

POSSIBLY USEFUL FORMULAS

$\sec^2 x = \tan^2 x + 1$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

$PV = nRT$

$F = \rho g A d$

$|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1}$

$M_n = \Delta x \left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \cdots + f\left(\frac{x_{n-1}+x_n}{2}\right)\right]$

$T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$

$\int \tan x\, dx = -\ln|\cos x| + C$

$|E_M| < \frac{K(b-a)^3}{24n^2} \quad (K \ge f''(x))$

$|E_T| < \frac{K(b-a)^3}{12n^2} \quad (K \ge f''(x))$

$\mathbf{n}\cdot(\mathbf{r}-\mathbf{r}_0) = 0$

$S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$

$|E_S| < \frac{K(b-a)^5}{180n^4} \quad (K \ge f^{(4)}(x))$

$g'(a) = \frac{1}{f'(g(a))}$

$\int_{n+1}^{\infty} f(x)\,dx \le s - s_n \le \int_n^{\infty} f(x)\,dx$

$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

$\sin 2x = 2\sin x \cos x$

$\text{Vol} = \int_a^b 2\pi\left[(f(x))^2 - (g(x))^2\right]\,dx$

$f(x) = y \iff f^{-1}(y) = x$

$\int f(x)\,dx = \int f(g(t))g'(t)\,dt$

SHOW WORK FOR FULL CREDIT NO CALCULATORS

1. **(16pts)** For each test and each series below, check exactly one box of the four possible choices. Recall that not all choices are valid for all tests. A series would fail the preconditions for the alternating series test, for example, if the the signs are not alternating. (No partial credit. You do not have to show work.)

	Divergence Test	**Integral Test**	**Geometric Series Test**	**Alternating Series Test**
$\sum_{n=1}^{\infty} \frac{(-1)^n 5^n}{7^n}$	☐ fails pre-conditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails pre-conditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails pre-conditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails pre-conditions ☐ diverges ☐ inconclusive ☐ converges
$\sum_{n=1}^{\infty} \frac{1}{n}$	☐ fails pre-conditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails pre-conditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails pre-conditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails pre-conditions ☐ diverges ☐ inconclusive ☐ converges
$\sum_{n=1}^{\infty} \frac{5 \cdot 4^{n-1}}{3^n}$	☐ fails pre-conditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails pre-conditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails pre-conditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails pre-conditions ☐ diverges ☐ inconclusive ☐ converges
$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$	☐ fails pre-conditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails pre-conditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails pre-conditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails pre-conditions ☐ diverges ☐ inconclusive ☐ converges

2. **(10pts)** Find the center, radius and the **interval** of convergence of the power series

$$\sum_{n=1}^{\infty}(-1)^n \frac{(x+2)^n}{n \cdot 7^n}.$$

3. **(10pts)** Use an appropriate test to determine the convergence of the series $\sum_{n=3}^{\infty} \frac{\ln n}{n}$. The preconditions of the test must be mentioned individually but do not have to be checked.

4. **(8pts)** Assume that $\sum_{n=1}^{\infty} a_n$ is a convergent series, with nth partial sum $s_n = 8 - \dfrac{3n}{4n+2}$.
What is the sum of the series?

5. **(8pts)** Determine whether the sequence $\{\ln(3n) - \ln(6n+3)\}_{n=1}^{\infty}$ converges or diverges.
Either compute the limit or the nature in which the sequence diverges.

6. **(8pts)** Determine the sum of the series $\sum_{n=0}^{\infty} \dfrac{(-1)^n 4^n}{7^{n+1}}$, or show that it diverges.

7. **(10pts)** For the two parts of this question, a complete technical proof is not necessary, but your answers must be justified.

   (a) For what values of r does the sequence $\left\{ \dfrac{n^r}{\sqrt{n}} \right\}_{n=1}^{\infty}$ converge?

   (b) For what values of t does the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^t}$ converge?

8. **(10pts)** Find an estimate for the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ which is within a tolerance of 10^{-4} of the actual sum. You do not have to completely simplify the estimate, but it must be totally clear what it is. (Hint: estimate, error, error bound, tolerance.)

9. **(10pts)** Determine whether the following series converge or diverge. You do not have to check the preconditions of the test you use, so long as those preconditions hold. Where appropriate, state how you are setting up the test, and the specific conclusion.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{4^n + 5^n}$ **(b)** $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{3^n n^2}$

10. **(a) (5pts)** Give an example of a series which converges but does not converge absolutely.

    (b) (5pts) Give an example of a series $\displaystyle\sum_{n=1}^{\infty} a_n$ such that $\displaystyle\sum_{n=1}^{\infty} \frac{a_n}{n!}$ converges but does not converge absolutely.