PRINT Last name:________________ First name:________________

Signature:________________ Student ID:________________

Math 152 Exam 2, Fall 2007

Instructions. You must show work in order to receive full credit. Partial credit is possible for work which makes positive progress toward the solution.

Conditions. ***No calculators***, notes, books, or scratch paper. By writing your name on the exam you certify that all work is your own, under penalty of all remedies outlined in the student handbook. Please do not talk until after leaving the room.

Time limit: 1 hour 15 minutes (strict).

NOTE: The topics may not be in order either of increasing difficulty or of the order they were covered in the course. Problems are **10 pts each** except where indicated.

POSSIBLY USEFUL FORMULAS

$\sec^2 x = \tan^2 x + 1$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

$PV = nRT$

$F = \rho g A d$

$|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1}$

$M_n = \Delta x \left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \cdots + f\left(\frac{x_{n-1}+x_n}{2}\right)\right]$

$T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$

$\int \tan x\, dx = -\ln|\cos x| + C$

$|E_M| < \frac{K(b-a)^3}{24n^2} \quad (K \ge f''(x))$

$|E_T| < \frac{K(b-a)^3}{12n^2} \quad (K \ge f''(x))$

$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$

$S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$

$|E_S| < \frac{K(b-a)^5}{180n^4} \quad (K \ge f^{(4)}(x))$

$g'(a) = \frac{1}{f'(g(a))}$

$\int_{n+1}^{\infty} f(x)\, dx \le s - s_n \le \int_n^{\infty} f(x)\, dx$

$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C$

$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

$\sin 2x = 2 \sin x \cos x$

$\text{Vol} = \int_a^b 2\pi\left[(f(x))^2 - (g(x))^2\right]\, dx$

$f(x) = y \iff f^{-1}(y) = x$

$\int f(x)\, dx = \int f(g(t))g'(t)\, dt$

SHOW WORK FOR FULL CREDIT NO CALCULATORS

1. Find the general solution to the differential equation

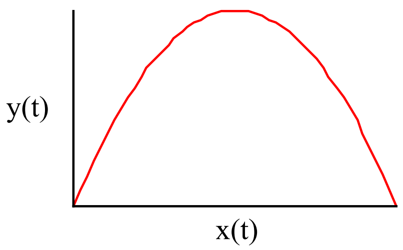
$$xy' + 2y = e^x \, ,$$

where $x > 0$ (this means assume $x > 0$ when deriving the general solution).

2. A pumpkin is fired so that its horizontal and vertical position at time t in seconds is given by $x(t)$ and $y(t)$, respectively, where

$$\begin{aligned} x(t) &= \frac{117}{22}\, t \\ y(t) &= \frac{44}{10}\, t - \frac{1}{10}\, t^2 \\ t &\geq 0. \end{aligned}$$

(a) Find the maximum height (vertical position) of the projectile.
(b) Find the time at which this height is achieved.
(c) If the ground is at height 0, determine the distance the pumpkin is launched.
(d) Is the angle of launch with respect to horizontal greater or less than 45^o?



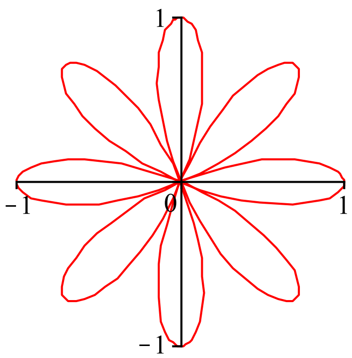
3. For this problem, use the differential equation for Newton's Law of Cooling:

$$\frac{dT}{dt} = k(T - T_s)\,,$$

where T is the temperature, t is time, T_s is the ambient temperature, and k is a constant.

Question: Water at temperature 50°F is placed in a freezer which is at 20°F. After 1 hour, the water is at temperature 40°F. How long does it take the water to freeze, that is, to reach 32°F? (Partial credit for writing the general solution without solving for it.)

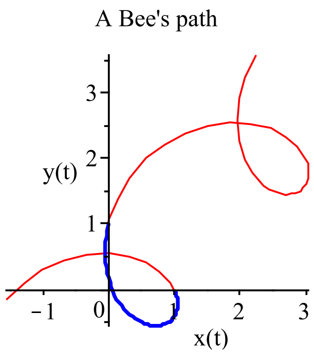
4. Find the area enclosed by one loop of the polar curve $r(\theta) = \cos(4\theta)$.



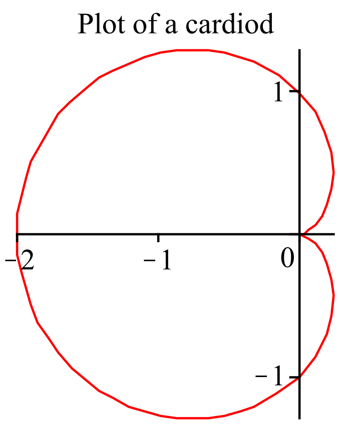
5. A bee buzzes along a path with parametric description

$$\begin{aligned} x(t) &= \frac{t}{2} + \cos\left(\frac{\pi t}{2}\right), \\ y(t) &= \frac{t}{2} - \sin\left(\frac{\pi t}{2}\right). \end{aligned}$$

Set up (**but do not integrate**) an integral representing the arc length of the portion of the path starting at $(x_1, y_1) = (1, 0)$ and ending at $(x_2, y_2) = (0, 1)$. The integrand should be a function of t which contains no derivatives.



6. A cardiod is given in polar form by the function $r(\theta) = 1 - \cos(\theta)$ for $0 \le \theta \le 2\pi$. Find the **horizontal** tangents to the plot of the cardiod, carefully describing the behavior at the cusp (at the pole). (Hint: It may be helpful to convert expressions to either all cosines or all sines.)



7. Find the area in the first quadrant bounded between the origin and the parametric curve given by

$$\begin{aligned} x(t) &= t^2 - 1\,, \\ y(t) &= e^{-2t^2+3} - 1\,, \\ t &\geq 0\,. \end{aligned} \tag{1}$$