PRINT Last name:______________ First name:______________

Signature:______________ Student ID:______________

# Math 152 Exam 2, Fall 2007

**Instructions.** You must show work in order to receive full credit. Partial credit is possible for work which makes positive progress toward the solution.

**Conditions.** ***No calculators***, notes, books, or scratch paper. By writing your name on the exam you certify that all work is your own, under penalty of all remedies outlined in the student handbook. Please do not talk until after leaving the room.

**Time limit:** 1 hour 15 minutes (strict).

**NOTE:** The topics may not be in order either of increasing difficulty or of the order they were covered in the course. Problems are **10 pts each** except where indicated.

## POSSIBLY USEFUL FORMULAS

$\sec^2 x = \tan^2 x + 1$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

$PV = nRT$

$F = \rho g A d$

$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$

$M_n = \Delta x \left[ f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \cdots + f\left(\frac{x_{n-1}+x_n}{2}\right) \right]$

$T_n = \frac{\Delta x}{2}\left[ f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n) \right]$

$\int \tan x\, dx = -\ln|\cos x| + C$

$|E_M| < \frac{K(b-a)^3}{24n^2} \quad (K \geq f''(x))$

$|E_T| < \frac{K(b-a)^3}{12n^2} \quad (K \geq f''(x))$

$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$

$S_n = \frac{\Delta x}{3}\left[ f(x_0) + 4f(x_1) + 2f(x_2) \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n) \right]$

$|E_S| < \frac{K(b-a)^5}{180n^4} \quad (K \geq f^{(4)}(x))$

$g'(a) = \frac{1}{f'(g(a))}$

$\int_{n+1}^{\infty} f(x)\, dx \leq s - s_n \leq \int_n^{\infty} f(x)\, dx$

$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C$

$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

$\sin 2x = 2 \sin x \cos x$

$\text{Vol} = \int_a^b 2\pi \left[ (f(x))^2 - (g(x))^2 \right] dx$

$f(x) = y \iff f^{-1}(y) = x$

SHOW WORK FOR FULL CREDIT NO CALCULATORS

1. Compute $\lim\limits_{x\to 0^+} (\cos(2x))^{1/x}$

2. (6pts) Evaluate $\displaystyle\int 3x^2e^{-2x}\,dx$ .

3. (6pts) Evaluate $\displaystyle\int \arcsin x\,dx$ .

4. Evaluate $\displaystyle\int_0^{\pi/2} \sin^3(5x)\,dx\,.$

5. Use an appropriate trig substitution of the form $x = g(\theta)$ to reexpress $\displaystyle\int_{3/2}^{3\sqrt{3}/2} \frac{\sqrt{9-x^2}}{x^2}\,dx$ **completely** in terms of $\theta$. **DO NOT** evaluate the resulting integral.

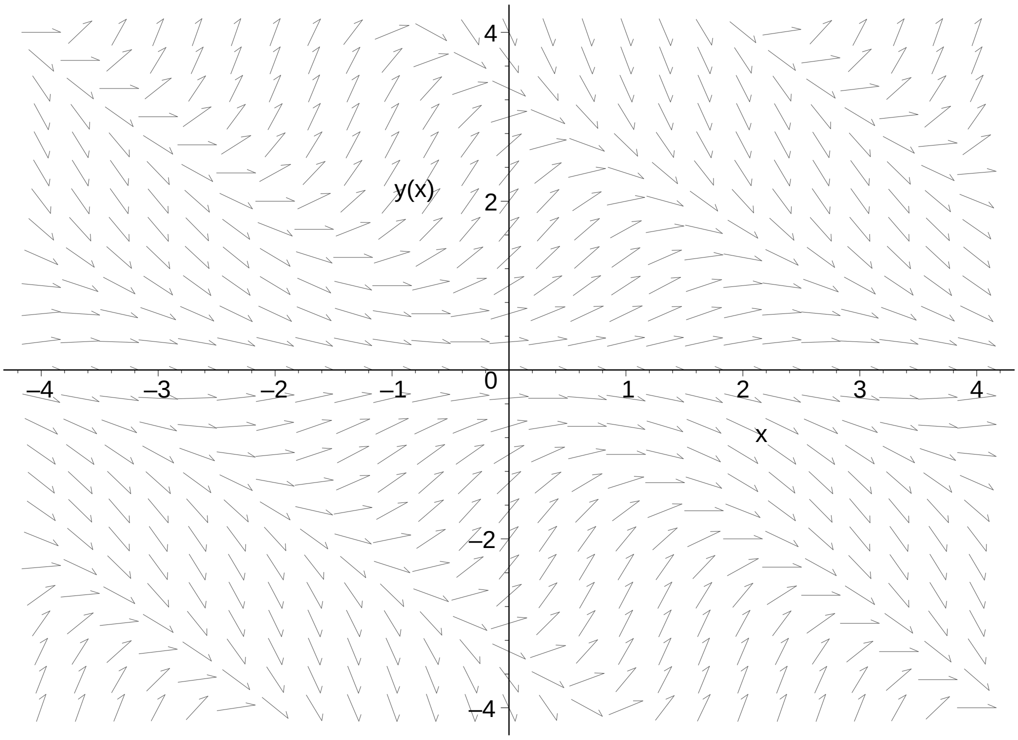
6. Compute the partial fraction decomposition for $\dfrac{x^2+9x-16}{(x-4)(x-1)^2}$. (Do not integrate.)

7. (12pts) Determine whether $\displaystyle\int_1^\infty \frac{dx}{x^{1/2} - \frac{1}{2}x^{1/4}}$ converges or diverges by comparing it to $\displaystyle\int_1^\infty \frac{dx}{x^p}$ for an appropriate value of $p$. (This requires computing the integral for your value of $p$).

8. A population $P(t)$ at time $t$ is modeled by the differential equation $\displaystyle\frac{dP}{dt} = 1.15P\left(1 - \frac{P}{6200}\right)$. For this problem, ignore negative values of $P$ ("population" is nonnegative).

   **(a)** For what values of $P$ is the population increasing?
   **(b)** For what values of $P$ is the population decreasing?
   **(c)** What are the equilibrium solutions?

9. The direction field corresponding to a differential equation is given here. Carefully sketch 3 particular solutions, passing through the points $(0, 0)$, $(2, 1)$, and $(0, -2)$, respectively.




10. (16pts)

**(a)** Find the general solution to the differential equation $\frac{1}{y}\frac{dy}{dt} = k$, where $k$ is a constant and $t$ is time. (Show steps.)

**(b)** From part **(a)**, find the particular solution satisfying $y(0) = 100$. Use this particular solution for parts **(c)** and **(d)**.

**(c)** Suppose $k$ is negative. Find the time time $T$ at which $y(T) = 50$.

**(d)** On the other hand, suppose $y(10) = 150$ and solve for $k$.