PRINT Last name:________________ First name:________________

Signature:________________ Student ID:________________

Math 152 Exam 1, Fall 2007

Instructions. You must show work in order to receive full credit. Partial credit is possible for work which makes positive progress toward the solution.

Conditions. ***No calculators***, notes, books, or scratch paper. By writing your name on the exam you certify that all work is your own, under penalty of all remedies outlined in the student handbook. Please do not talk until after leaving the room.

Time limit: 1 hour 15 minutes (strict).

NOTE: The topics may not be in order either of increasing difficulty or of the order they were covered in the course.

POSSIBLY USEFUL FORMULAS

$\sec^2 x = \tan^2 x + 1$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

$PV = nRT$

$F = \rho g A d$

$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$

$M_n = \Delta x \left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \cdots + f\left(\frac{x_{n-1}+x_n}{2}\right)\right]$

$T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$

$\int \tan x \, dx = -\ln|\cos x| + C$

$|E_M| < \frac{K(b-a)^3}{24n^2} \quad (K \geq f''(x))$

$|E_T| < \frac{K(b-a)^3}{12n^2} \quad (K \geq f''(x))$

$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$

$S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$

$|E_S| < \frac{K(b-a)^5}{180n^4} \quad (K \geq f^{(4)}(x))$

$g'(a) = \frac{1}{f'(g(a))}$

$\int_{n+1}^{\infty} f(x)\, dx \leq s - s_n \leq \int_n^{\infty} f(x)\, dx$

$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C$

$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

$\sin 2x = 2 \sin x \cos x$

$\text{Vol} = \int_a^b 2\pi \left[(f(x))^2 - (g(x))^2\right] dx$

$f(x) = y \iff f^{-1}(y) = x$

SHOW WORK FOR FULL CREDIT NO CALCULATORS

1. Find the inverse function, including domain and range, for the function $f(x) = \sqrt{x-4}$ with domain $4 \leq x \leq 13$.

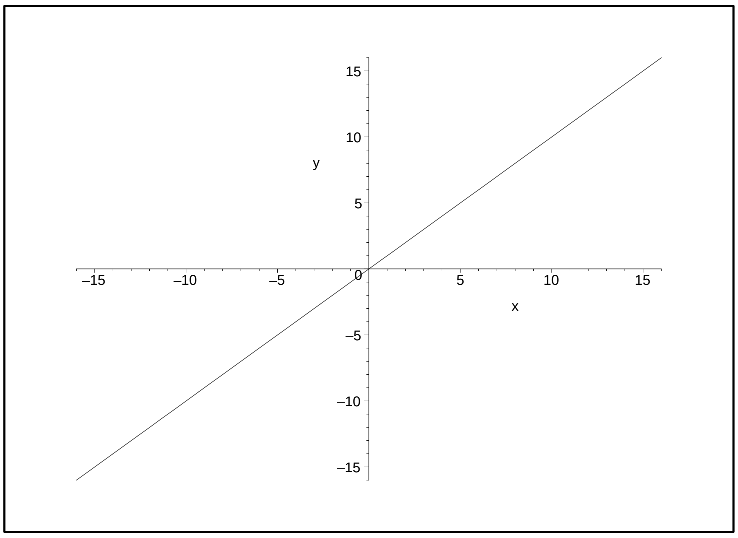
2. In order to stop the doomsday countdown of a computer on a certain island out of B. F. Skinner's nightmares, you must type in the derivative of a Lambert-type function $g(x)$ at $x = e$. All you know is that $g(x)$ is the inverse function of $f(x) = x \cdot \ln x$, where the domain of f is $1 \leq x < \infty$. Save the island by computing $g'(e)$.

3. If $f(x) = \dfrac{x-4}{x+4}$ and $g(x)$ is the inverse of $f(x)$, find $g(0)$.

4. Let $f(x) = \dfrac{1}{\sqrt{4-2x}}$.
(a) What is the largest possible domain for $f(x)$? (Show calculation.)
(b) Prove that $f(x)$ is one-to-one on this domain by considering $f'(x)$. (Half-credit for a correct sketch plus the horizontal line test.)

5. Simplify the expressions **(a)** $\cos\left(\arcsin\frac{1}{2}\right)$ and **(b)** $\arccos\left(\cos\frac{8\pi}{6}\right)$.

6. Let $f(x) = (\sqrt{3})^x$.
   (a) What is the inverse of $f(x)$?
   (b) Plot and label $f(x)$ and its inverse on the axes below, and label at least three **pairs of points** (6 total), where a pair of points has one point on the graph of $y = f(x)$ and the corresponding point on the graph of the inverse.



7. Evaluate the integral $\displaystyle\int_0^{\pi/2} \sin x \cdot 3^{\cos x}\,dx$.

8. Differentiate the function $f(x) = x \cdot e^{\sin(x)}$.

9. Compute $\lim_{x\to\infty} (x^2+x)e^{-x}$.

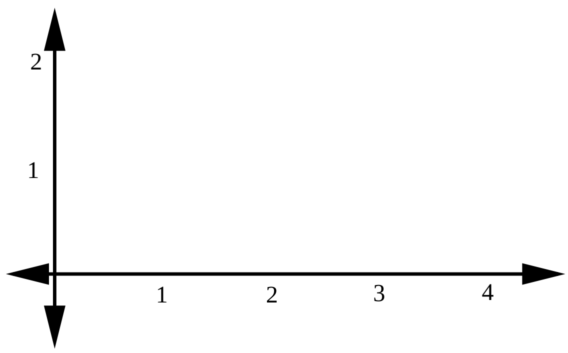
10. Compute $\lim_{x\to 0^+} \dfrac{\arctan(x)}{x}$.

11. Compute $\lim_{x\to \frac{\pi}{2}^-} (\tan x)^{\left(\frac{2x}{\pi}\right)}$.

12. Compute the derivative of the function $f(x) = (\arcsin x)^{2x}$.

13. Evaluate the integral $\displaystyle\int \frac{\sec^2 x}{\tan x}\, dx\,.$

14. Recall that by definition, $\ln x := \displaystyle\int_1^x \frac{1}{t}\, dt\,.$ On the axes below, illustrate $\ln 4 - \ln 3$ as an area, and label **all relevant quantities**.



15. Using the Laws of Logarithms write the following quantity in the form $\ln(\cdot)$ (natural log of a single argument): $2\ln\cos(x) + 5\ln(x-4) - 10\ln(z+w)$

16. Differentiate the function $f(x) = \dfrac{4}{3}\,(\ln x)^3\,.$

(Extra workspace – **clearly label problem numbers and circle answers!!**)

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