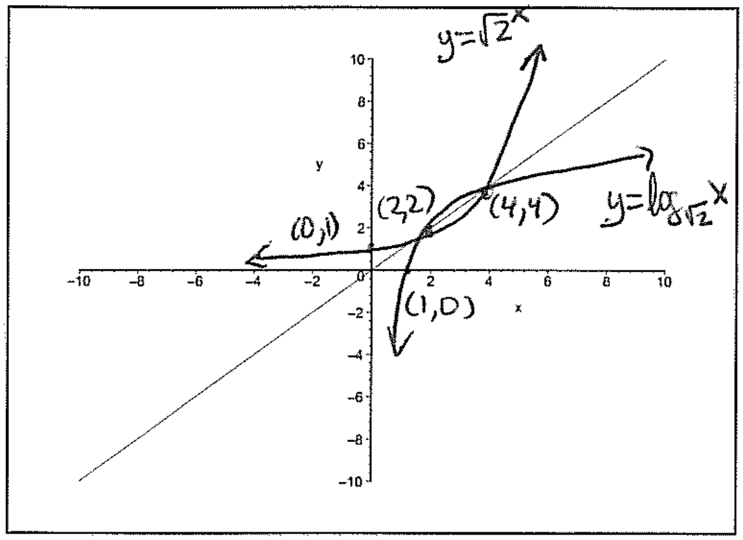
SHOW WORK FOR FULL CREDIT NO CALCULATORS

1. Let $f(x) = (\sqrt{2})^x$.

2 (a) What is the inverse of $f(x)$?

0 (b) Plot and label $f(x)$ and its inverse on the axes below, and label at least three **pairs of points** (6 total), where a pair of points has one point on the graph of $y = f(x)$ and the corresponding point on the graph of the inverse.



(a) $f^{-1}(x) = \log_{\sqrt{2}} x$

(b) $f(0) = 1$

$f(2) = 2$

$f(4) = 4$

2. Differentiate the function $f(x) = x \cdot 3^{\cos x}$.

$$f'(x) = \boxed{x\, 3^{\cos x} \ln 3 (-\sin x) + (1)\, 3^{\cos x}}$$

$$= \boxed{3^{\cos x}(-x \ln 3 \sin x + 1)}$$

3. Evaluate the integral $\int x^2 \cdot e^{x^3}\, dx$.

$u = x^3$ (1)

$du = 3x^2\, dx$ (1)

$\frac{du}{3} = x^2\, dx$ (1)

$\int \frac{e^u}{3}\, du$ (1)

$= \frac{e^u}{3} + C$ (1)

$= \boxed{\frac{e^{x^3}}{3} + C}$ (1)

4. Simplify the expressions **(a)** $\sin\left(\arccos \frac{1}{2}\right)$ and **(b)** $\arcsin\left(\sin \frac{7\pi}{6}\right)$.

(a) $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\sin\left(\frac{\pi}{3}\right) = \boxed{\frac{\sqrt{3}}{2}}$

(b) $\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$

$\sin^{-1}\left(-\frac{1}{2}\right) = \boxed{\frac{-\pi}{6}}$

6

5. Find the inverse function, including domain and range, for the function $f(x) = 9 - x^2$ with domain $0 \le x \le 3$.

range of $f(x)$: $0 \le x \le 3$
$0 \le x^2 \le 9$
$0 \ge -x^2 \ge -9$
$9 \ge 9-x^2 \ge 0$
so range is $\boxed{[0,9]}$

find $f^{-1}(x)$: $y = 9-x^2$
$x = 9-y^2$
$x-9 = -y^2$
$9-x = y^2$
$\pm\sqrt{9-x} = y$
range $[0,3] \Rightarrow$ take + root
$\boxed{f^{-1}(x) = \sqrt{9-x}}$

$f^{-1}(x)$ has $\boxed{\text{domain } [0,9] \quad \text{range } [0,3]}$

6

6. If $f(x) = \dfrac{x+3}{x-3}$ and $g(x)$ is the inverse of $f(x)$, find $g(0)$.

$g(0) = x \Leftrightarrow f(x) = 0$.
Solve $f(x) = 0$
$\dfrac{x+3}{x-3} = 0$
$x+3 = 0$
$x = -3$.
so $\boxed{g(0) = -3}$

7

7. In order to stop the doomsday countdown of a computer on a certain island out of B. F. Skinner's nightmares, you must type in the derivative of the Lambert W-function $g(x)$ at $x = e$. All you know is that $g(x)$ is the inverse function of $f(x) = x \cdot e^x$, where the domain of f is $0 \le x < \infty$. Save the island by computing $g'(e)$.

coverFormula: $g'(e) = \dfrac{1}{f'(g(e))}$

$g(e) = x \Leftrightarrow f(x) = e$
Solve $xe^x = e$ 2

$x = 1$. 1
so $g(e) = 1$.

$f'(x) = xe^x + e^x$ 2
so $g'(e) = \dfrac{1}{f'(1)} = \dfrac{1}{1e^1 + e^1}$
$= \boxed{\dfrac{1}{2e}}$ 0

7

8. Let $f(x) = e^{1/x}$ (also written as $f(x) = \exp\left(\dfrac{1}{x}\right)$).

(a) What is the largest possible domain for $f(x)$? (Show calculation.)
(b) Prove that $f(x)$ is one-to-one on this domain by considering $f'(x)$. (Half-credit for a correct sketch plus the horizontal line test.)

$\boxed{3}$ (a) require $x \ne 0$
so $\boxed{\text{domain } (-\infty,0)\cup(0,\infty)}$

$\boxed{4}$ (b) $f'(x) = e^{1/x}\left(-\dfrac{1}{x^2}\right) = -\dfrac{e^{1/x}}{x^2}$.

when $x \ne 0$, $-e^{1/x} < 0$
$x^2 > 0$.

9. Compute the derivative of the function $f(x) = (1+x^2)^{\arcsin x}$.

$y = (1+x^2)^{\sin^{-1}x}$

$\ln y = \ln\left[(1+x^2)^{\sin^{-1}x}\right]$

$\ln y = \sin^{-1}x \, \ln(1+x^2)$

$\frac{y'}{y} = \frac{1}{\sqrt{1-x^2}} \ln(1+x^2) + \sin^{-1}x \, \frac{2x}{1+x^2}$

$y' = (1+x^2)^{\sin^{-1}x} \left(\frac{\ln(1+x^2)}{\sqrt{1-x^2}} + \frac{2x \sin^{-1}x}{1+x^2} \right)$

10. Compute $\lim_{x\to\infty} x \cdot \arctan(1/x)$.

Form: $\infty \cdot 0$ IF

$= \lim_{x\to\infty} \frac{\tan^{-1}(1/x)}{\frac{1}{x}}$

$\overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x^2}}\left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}}$

$= \lim_{x\to\infty} \frac{1}{1+\frac{1}{x^2}} = \boxed{1}$

11. Compute $\lim_{x\to\infty} \frac{(\ln x)^2}{x^2}$.

Form $\frac{\infty}{\infty}$ IF

$\overset{L'H}{=} \lim_{x\to\infty} \frac{2\ln x \left(\frac{1}{x}\right)}{2x}$

$= \lim_{x\to\infty} \frac{\ln x}{x^2}$

Form $\frac{\infty}{\infty}$

$\overset{L'H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{2x}$

$= \lim_{x\to\infty} \frac{1}{2x^2} = \boxed{0}$

12. Compute $\lim_{x\to 1} \left(\frac{1}{x}\right)^{(\ln x)}$.

Form 1^0. Not IF.

$= \lim_{x\to 1} \exp\left(\ln\left[\left(\frac{1}{x}\right)^{\ln x}\right]\right)$

$= \lim_{x\to 1} \exp\left(\ln x \ln \frac{1}{x}\right)$

$= \exp\left(\lim_{x\to 1} \ln x \ln \frac{1}{x}\right)$

$= \exp(0) = \boxed{1}$

13. Differentiate the function $f(x) = \ln(\tan x)$.

$$f'(x) = \boxed{\frac{\sec^2 x}{\tan x}}$$

14. Evaluate the integral $\displaystyle\int_1^e \frac{4(\ln x)^2}{x}\,dx$.

$u = \ln x$

$du = \dfrac{dx}{x}$

$$\int_{x=1}^{e} 4u^3\,du$$

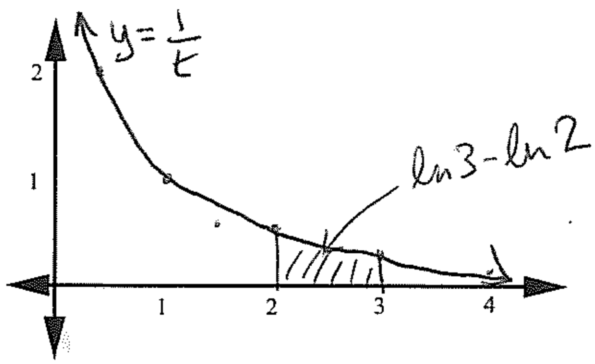
$$= \frac{4u^3}{3}\Bigg|_{x=1}^{e}$$

$$= \frac{4(\ln x)^3}{3}\Bigg|_1^e$$

$$= \frac{4(\ln e)^3}{3} - \frac{4(\ln 1)^3}{3}$$

$$= \boxed{\frac{4}{3}}$$

15. Recall that by definition, $\ln x := \displaystyle\int_1^x \frac{1}{t}\,dt$. On the axes below, illustrate $\ln 3 - \ln 2$ as an area, and label **all relevant quantities**.



16. Using the Laws of Logarithms write the following quantity in the form $\ln(\cdot)$ (natural log of a single argument): $2\ln\cos(x) + 5\ln(x-4) - 10\ln(z+w)$

$$= \ln\cos^2 x + \ln(x-4)^5 - \ln(z+w)^{10}$$

$$= \boxed{\ln\left[\frac{(\cos^2 x)(x-4)^5}{(z+w)^{10}}\right]}$$