PRINT Last name:________________ First name:________________________

Signature:______________________ Student ID:______________________

Math 152 Final, Fall 2006

Instructions. You must show work in order to receive full credit. Partial credit is possible for work which makes positive progress toward the solution.

Conditions. No calculators, notes, books, or scratch paper. By writing your name on the exam you certify that all work is your own, under penalty of all remedies outlined in the student handbook. Please do not talk until after leaving the room.

Time limit: 2 hours (strict).

NOTE: The topics may not be in order either of increasing difficulty or of the order they were covered in the course. Form A1.

POSSIBLY USEFUL FORMULAS

$\sec^2 x = \tan^2 x + 1$
$\cos^2 x = \frac{1+\cos 2x}{2}$
$\sin^2 x = \frac{1-\cos 2x}{2}$
$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$
$\int \tan x\, dx = -\ln|\cos x| + C$
If $|f^{(n+1)}(x)| \leq M$ for $|x-a| \leq d$, then $|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$ for $|x-a| \leq d$.
$g'(a) = \frac{1}{f'(g(a))}$
$\int_{n+1}^{\infty} f(x)\, dx \leq s - s_n \leq \int_n^{\infty} f(x)\, dx$
$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$
$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$
$\sin 2x = 2 \sin x \cos x$
$\text{Vol} = \int_a^b 2\pi \left[(f(x))^2 - (g(x))^2\right] dx$
$f(x) = y \Leftrightarrow f^{-1}(y) = x$

SHOW WORK FOR FULL CREDIT NO CALCULATORS

1. **(7pts)** Compute the 4th degree Taylor Polynomial $T_4(x)$ centered at 1 for the function $f(x) = \ln x$.

2. **(7pts)** Compute the Taylor Series $T(x)$ centered at 3 for $f(x) = e^x$.

3. **(7pts)** By converting the integrand into a power series, evaluate the indefinite integral

$$\int x \cos\left(x^3\right) dx \, .$$

4. **(7pts)** Use the 3rd degree Taylor Polynomial

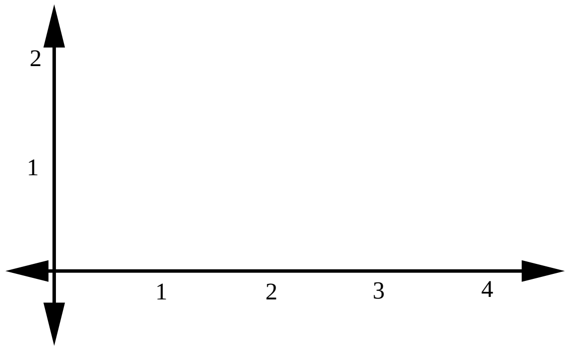
$$T_3(x) \;=\; \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) - \frac{1}{4}\left(x - \frac{\pi}{6}\right)^2 - \frac{\sqrt{3}}{12}\left(x - \frac{\pi}{6}\right)^3$$

to estimate $\sin(35^o) = \sin(7\pi/36)$. Find an upper bound on the error in the approximation using Taylor's Inequality (the Taylor Remainder bound). Do not attempt to convert expressions into decimal form.

5. **(5pts)** Find the limit $\lim_{x \to 0^+} x^2 \cdot (\ln x)^2$.

6. **(5pts)** Compute the derivative of the function $f(x) = (1+2x)^{\arctan x}$.

7. **(4pts)** Recall that by definition, $\ln x := \int_1^x \frac{1}{t}\, dt$. On the axes below, illustrate $\ln\left(\frac{1}{2}\right)$ as an area, and label all relevant quantities. **Explain** how we can understand from the illustration that $\ln\left(\frac{1}{2}\right)$ is negative.



8. **(8pts)** Evaluate the integral $\displaystyle\int_1^\infty x^3 \cdot e^{-2x}\, dx$.

9. **(8pts)** Evaluate the integral $\displaystyle\int \frac{1}{(1+x^2)^2}\, dx$.

10. **(7pts)** Polonium-210 (symbol Po, atomic number 84), the radioactive compound used recently to kill Russian dissident Alexander Litvinenko, has a half-life of 138 days.

(a) Write down the general solution for a radioactive decay equation, or re-derive it from the differential equation $\dfrac{dy}{dt} = ky$, where t is in days.

(b) Write down the particular solution corresponding to an initial amount of 3 (grams) at time $t = 0$.

(c) Determine the value of k in the governing differential equation $\dfrac{dy}{dt} = ky$.

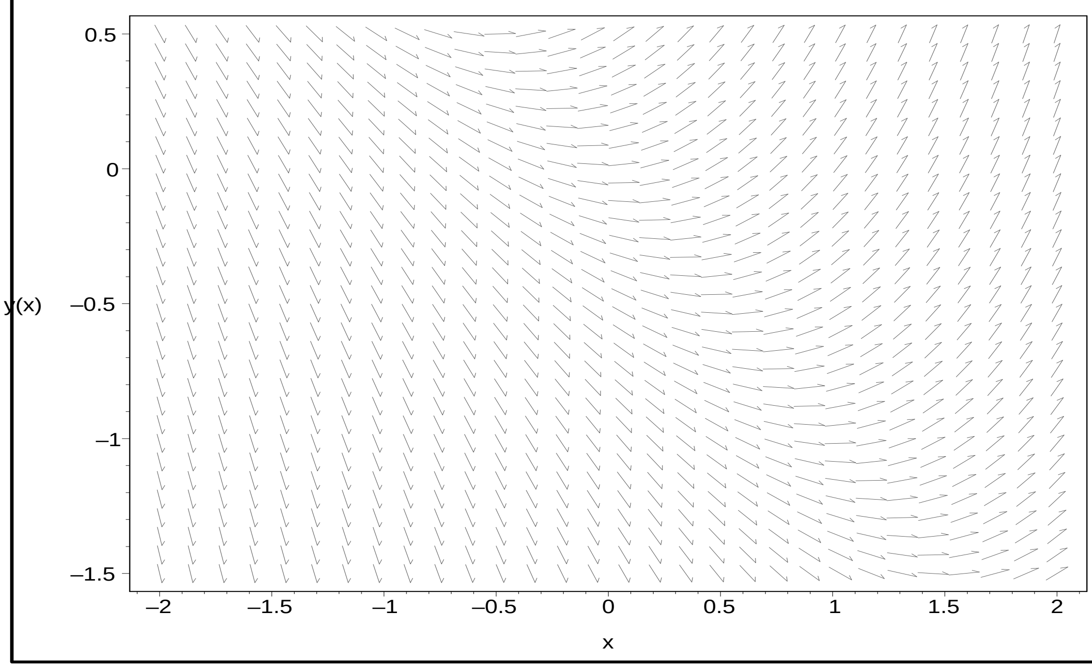
(d) Compute the time at which only 40% of the initial amount remains.

11. **(7pts)** The slope field below is for the differential equation $y' = x + y$.

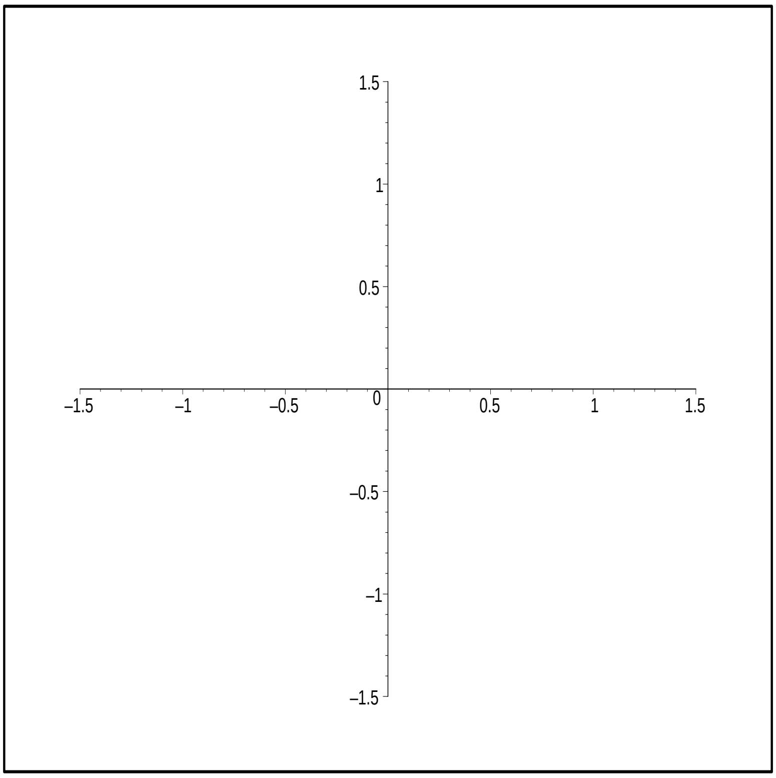
(a) Approximate $y(2)$ assuming that $y(0) = -\frac{1}{2}$ by sketching the particular solution on the slope field.

(b) Approximate $y(2)$ assuming that $y(0) = -\frac{1}{2}$ by using Euler's method with step size $\Delta x = 0.5$ (sketch Euler's method on the slope field).

(**Hint:** In both cases, it is possible to check your answer exactly.)



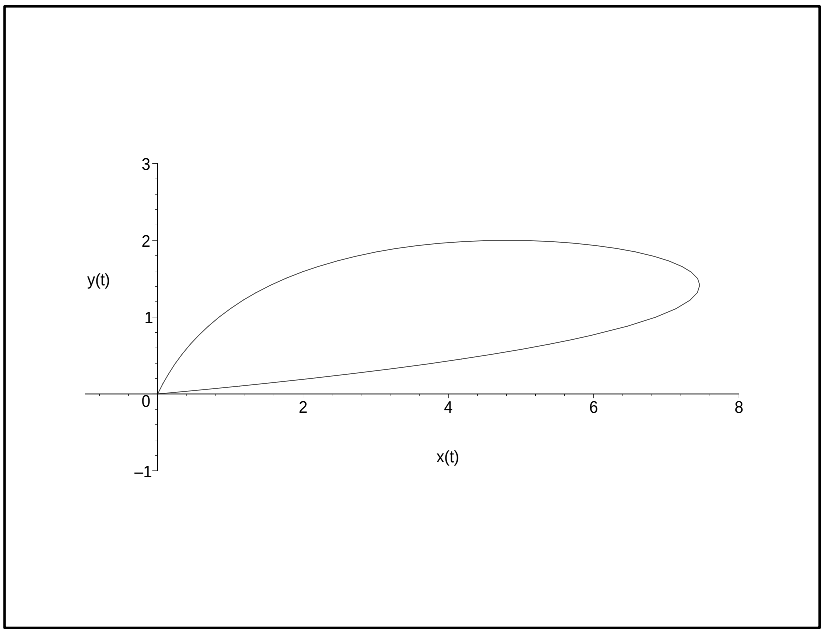
12. **(8pts)** Sketch the plot of the **polar** function $r(\theta) = \cos(4\theta)$ on the range $\frac{\pi}{8} \leq \theta \leq \frac{7\pi}{8}$.



13. **(8pts)** The plot below is generated by a particle following the parametric equations

$$x(t) = e^t \cdot \sin t$$
$$y(t) = 2\sin t$$
$$0 \le t \le \pi.$$

Determine **exactly** the maximum horizontal position $x(t)$ of the particle, and the value of t for which this occurs.



14. **(6pts)** Use an appropriate test to determine the convergence of the series $\sum_{n=0}^{\infty} \frac{\ln n}{n}$.

15. **(6pts)** Determine exactly the entire interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{n^3(x-2)^n}{6 \cdot 3^n}.$$