PRINT Last name:_____ First name:_____

Signature:_____ Student ID:_____

Math 152 Exam 4, Fall 2006

Instructions. You must show work in order to receive full credit. Partial credit is possible for work which makes positive progress toward the solution.

Conditions. No calculators, notes, books, or scratch paper. By writing your name on the exam you certify that all work is your own, under penalty of all remedies outlined in the student handbook. Please do not talk until after leaving the room.

Time limit: 1 hour 15 minutes (strict).

NOTE: The topics may not be in order either of increasing difficulty or of the order they were covered in the course. Form A1.

POSSIBLY USEFUL FORMULAS

$\sec^2 x = \tan^2 x + 1$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

$\int \tan x \, dx = -\ln|\cos x| + C$

$g'(a) = \frac{1}{f'(g(a))}$

$\int_{n+1}^{\infty} f(x)\,dx \le s - s_n \le \int_n^{\infty} f(x)\,dx$

$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

$\sin 2x = 2\sin x \cos x$

$\text{Vol} = \int_a^b 2\pi\left[(f(x))^2 - (g(x))^2\right]\,dx$

$f(x) = y \;\Leftrightarrow\; f^{-1}(y) = x$

SHOW WORK FOR FULL CREDIT NO CALCULATORS

1. For a certain series $\sum_{n=1}^{\infty} a_n$, the sequence of partial sums is

$$\{s_n\}_{n=1}^{\infty} = \left\{1 - 4^{-n}\right\}_{n=1}^{\infty} .$$

(a) (4pts) Find a closed formula for a_n.
(b) (4pts) Determine whether or not the series converges, and if so, find the sum.

2. **(a) (5pts)** For what values of q does the series $\sum_{n=1}^{\infty} n^q$ converge?

(b) (5pts) For what values of s does the series $\sum_{n=0}^{\infty} \frac{5 \cdot 2^n}{s^n}$ converge?

3. **(10pts)** Starting with power series representation

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad \text{when } -1 < x < 1,$$

find a power series representation for $\dfrac{x^4}{1+x^3}$ and determine for exactly which values of x the representation is valid.

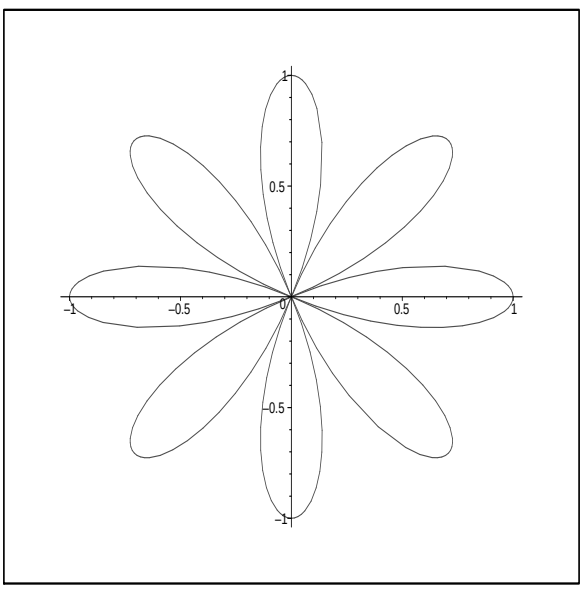
4. **(10pts)** Determine for *exactly* which values of x the power series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n x^n}{n}$ converges.

5. **(10pts)** For each test and each series below, check exactly one box of the four possible choices. Recall that not all choices are valid for all tests. A series would fail the preconditions for the alternating series test, for example, if the the signs are not alternating. (No partial credit. You do not have to show work.)

	Divergence Test	**Integral Test**	**Geometric Series Test**	**Alternating Series Test**
$\sum_{n=1}^{\infty} \frac{1}{n}$	☐ fails preconditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails preconditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails preconditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails preconditions ☐ diverges ☐ inconclusive ☐ converges
$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$	☐ fails preconditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails preconditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails preconditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails preconditions ☐ diverges ☐ inconclusive ☐ converges
$\sum_{n=1}^{\infty} \frac{5 \cdot 4^{n-1}}{3^n}$	☐ fails preconditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails preconditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails preconditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails preconditions ☐ diverges ☐ inconclusive ☐ converges
$\sum_{n=1}^{\infty} \frac{(-1)^n 5^n}{7^n}$	☐ fails preconditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails preconditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails preconditions ☐ diverges ☐ inconclusive ☐ converges	☐ fails preconditions ☐ diverges ☐ inconclusive ☐ converges

6. **(10pts)** Given the function $f(x) = \dfrac{x}{9} + \dfrac{1}{x}$, we define the sequence $\{a_n\}_{n=k}^{\infty}$ by setting $a_n = f(n)$.
(a) Find k as small as possible so that $\{a_n\}_{n=k}^{\infty}$ is an *increasing* sequence. (Hint:use $f'(x)$.)
(b) Is the sequence bounded? If so, give valid bounds. If not, compute $\lim_{n\to\infty} a_n$.

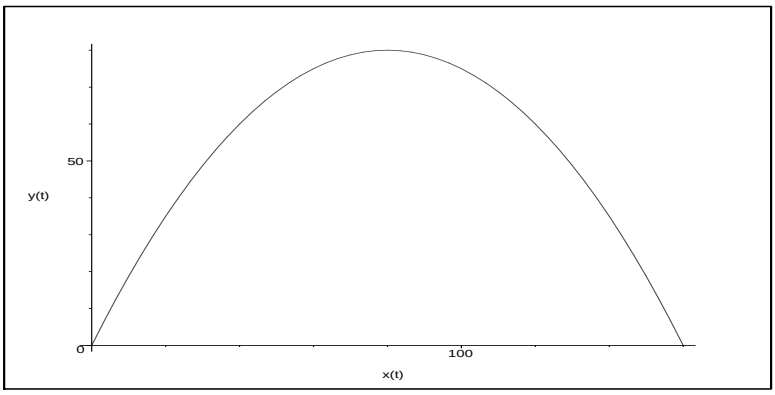
7. **(10pts)** Find the area enclosed by one loop of the polar curve $r(\theta) = \cos(4\theta)$.



8. **(10pts)** A projectile is fired so that its horizontal and vertical position at time t in seconds is given by $x(t)$ and $y(t)$, respectively, where

$$\begin{aligned} x(t) &= 20t \\ y(t) &= 40t - 5t^2 \\ t &\geq 0. \end{aligned}$$

(a) Find A and B so that the projectile has height at least 60 exactly when $A \leq t \leq B$.
(b) Set up but **do not integrate** an integral representing the arc length traced out by the projectile when its height is at least 60. Write "A" and "B" in place of the quantities found in part (a).



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