PRINT Last name:__________________ First name:__________________________

Signature:__________________________ Student ID:__________________________

Math 152 Exam 2, Fall 2006

Instructions. You must show work in order to receive full credit. Partial credit is possible for work which makes positive progress toward the solution.

Conditions. No calculators, notes, books, or scratch paper. By writing your name on the exam you certify that all work is your own, under penalty of all remedies outlined in the student handbook. Please do not talk until after leaving the room.

Time limit: 1 hour 15 minutes (strict).

NOTE: The topics may not be in order either of increasing difficulty or of the order they were covered in the course. **Problems are 10 points each**. Form A1.

POSSIBLY USEFUL FORMULAS

$\sec^2 x = \tan^2 x + 1$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

$PV = nRT$

$F = \rho g A d$

$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$

$M_n = \Delta x \left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \cdots + f\left(\frac{x_{n-1}+x_n}{2}\right)\right]$

$T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$

$\int \tan x \, dx = -\ln|\cos x| + C$

$|E_M| < \frac{K(b-a)^3}{24n^2} \quad (K \geq f''(x))$

$|E_T| < \frac{K(b-a)^3}{12n^2} \quad (K \geq f''(x))$

$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$

$S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$

$|E_S| < \frac{K(b-a)^5}{180n^4} \quad (K \geq f^{(4)}(x))$

$g'(a) = \frac{1}{f'(g(a))}$

$\int_{n+1}^{\infty} f(x)\,dx \leq s - s_n \leq \int_n^{\infty} f(x)\,dx$

$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

$\sin 2x = 2 \sin x \cos x$

$\mathrm{Vol} = \int_a^b 2\pi \left[(f(x))^2 - (g(x))^2\right]\,dx$

$f(x) = y \iff f^{-1}(y) = x$

SHOW WORK FOR FULL CREDIT NO CALCULATORS

1. Find the exact value of the following:

   (a) $\tan^{-1}(\tan(3\pi/4))$ **(b)** $\sin(\cos^{-1}(2/3))$

2. Evaluate the integral $\displaystyle\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx$.

3. Evaluate the integral $\displaystyle\int 2x^2 e^{3x}\,dx$.

4. Find A, B, and C in the partial fraction decomposition

$$\frac{x^2+x+8}{(x-2)(x^2+3)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+3}\,.$$

5. Evaluate the integral $\displaystyle\int \frac{x-1}{(x-2)^2}\,dx\,.$

6. Find a function $f(x)$ such that $\displaystyle\int_1^\infty f(x)\,dx = \infty$, but $\displaystyle\int_1^\infty \frac{f(x)}{\sqrt{x}}\,dx$ converges. Give the exact value of the second integral or otherwise prove it converges.

7. Use integration by parts to evaluate the integral $\displaystyle\int \tan^{-1} x \, dx$.

8. Recall that the degree 1 finite Fourier approximation of $f(x)$ is $g(x) = a + b\sin x + c\cos x$, where

$$a = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx\,, \quad b = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(x)\,dx\,, \text{ and } \quad c = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(x)\,dx\,.$$

When $f(x) = x^3$, determine whether a, b, and c are positive, negative, or 0, and **give a justification for each**. Circle one answer for each of a, b, and c.

a: $< 0 \quad = 0 \quad > 0$ **b:** $< 0 \quad = 0 \quad > 0$ **c:** $< 0 \quad = 0 \quad > 0$

9. Evaluate the integral $\displaystyle\int \sec^4 x \, \tan^2 x \, dx\,.$

10. Evaluate the integral $\displaystyle\int_0^3 \sqrt{9-x^2}\, dx$ in two ways: **(i)** by direct evaluation, and **(ii)** by interpreting the integral as a geometric area.