PRINT Last name:________________ First name:____________________

Signature:____________________ Student ID:____________________

Math 152 Exam 1, Fall 200~~5~~ 6

Instructions. *Part I* is multiple choice. There will be no partial credit. Clearly indicate your answer, especially if you change an answer. Problems with two indicated answers will receive no credit.
Part II is work-out problems. You must show work in order to receive full credit. Partial credit is possible for work which makes positive progress toward the solution.
Part III consists of the more conceptual problems; otherwise the instructions are the same as Part II.

Conditions. No calculators, notes, books, or scratch paper. By writing your name on the exam you certify that all work is your own, under penalty of all remedies outlined in the TAMU student rules. Please do not talk until after leaving the room.

Time limit: 2 hours (strict).

NOTE: The topics may not be in order either of increasing difficulty or of the order they were covered in the course.

POSSIBLY USEFUL FORMULAS

$\sec^2 x = \tan^2 x + 1$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

$PV = nRT$

$F = \rho g A d$

$|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1}$

$M_n = \Delta x\left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \cdots + f\left(\frac{x_{n-1}+x_n}{2}\right)\right]$

$T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$

$\int \tan x\, dx = -\ln|\cos x| + C$

$|E_M| < \frac{K(b-a)^3}{24n^2} \quad (K \ge f''(x))$

$|E_T| < \frac{K(b-a)^3}{12n^2} \quad (K \ge f''(x))$

$\mathbf{n}\cdot(\mathbf{r}-\mathbf{r}_0) = 0$

$S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2)\cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$

$|E_S| < \frac{K(b-a)^5}{180n^4} \quad (K \ge f^{(4)}(x))$

$g'(a) = \frac{1}{f'(g(a))}$

$\int_{n+1}^{\infty} f(x)\,dx \le s - s_n \le \int_n^{\infty} f(x)\,dx$

$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

$\frac{1}{1+x^2} = \sum_{n=0}^{\infty}(-1)^n x^{2n}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

$\sin 2x = 2\sin x\cos x$

$\text{Vol} = \int_a^b 2\pi\left[(f(x))^2 - (g(x))^2\right]\,dx$

$f(x) = y \iff f^{-1}(y) = x$

SHOW WORK FOR FULL CREDIT NO CALCULATORS

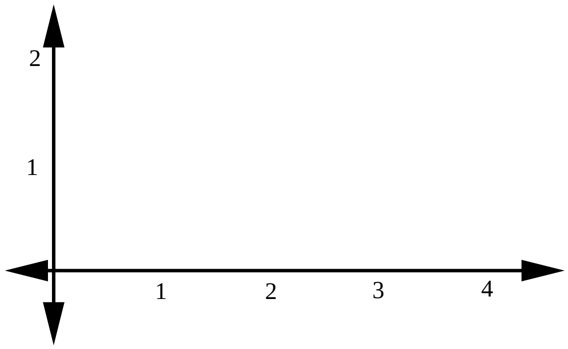
1. If $f(x) = x + \ln x$ and $g(x)$ is the inverse of $f(x)$, find $g(1)$.

2. Let $f(x) = \sqrt{2x - 4}$.
(a) What is the largest possible domain for $f(x)$? (Show calculation.)
(b) Prove that $f(x)$ is one-to-one on this domain by considering $f'(x)$. (Half-credit for a correct sketch plus the horizontal line test.)

3. Find a formula for the inverse of the function $f(x) = \dfrac{3x - 5}{4x + 5}$.

4. If $f(x) = x + e^x$ and $g(x)$ is the inverse of $f(x)$, find the **derivative of** g **at 1**, $g'(1)$.

5. Recall that by definition, $\ln x := \displaystyle\int_1^x \frac{1}{t}\,dt$. On the axes below, illustrate $\ln(2.5)$ as an area, and label **all relevant quantities**.

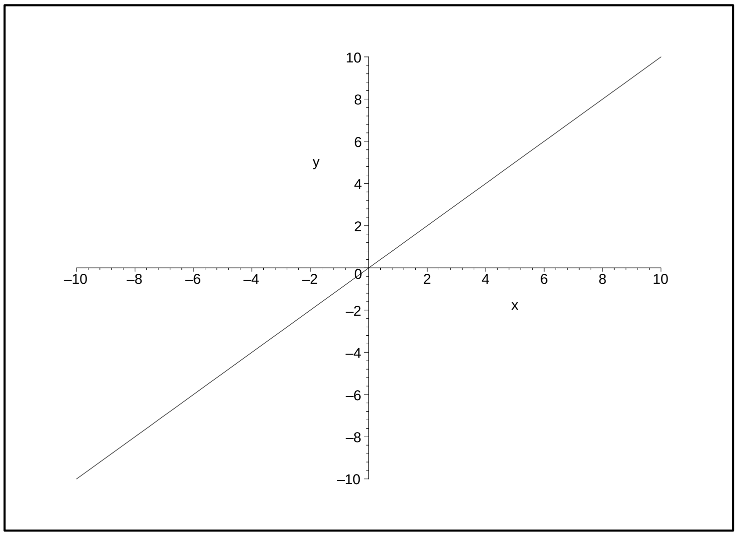


6. Completely expand the following expression using the Laws of Logarithms: $\ln\left(\dfrac{(\sin x)^2}{(x+2)^4(z-y)^5}\right)$

7. Differentiate the function $f(x) = \ln(\tan x)$.

8. Evaluate the integral $\displaystyle\int \frac{(\ln x)^3}{x}\,dx$.

9. Let $f(x) = \log_2 x$.
(a) What is the inverse of $f(x)$?
(b) Plot $f(x)$ and its inverse on the axes below, and label at least three **pairs of points** (6 total), where a pair of points has one point on the graph of $y = f(x)$ and the corresponding point on the graph of the inverse.



10. Differentiate the function $f(x) = x \cdot e^{\sin x}$.

11. Evaluate the integral $\int_0^1 x \cdot 3^{x^2}$.

12. Simplify the expression $\ln\left(e^{\sin x - x}e^y\right)$.

13. Find the limit $\lim_{x\to\infty} \dfrac{e^{x^2}}{x^4}$.

14. Find the limit $\lim_{x\to 1^+} \dfrac{\log_2(x^2+1)}{(x-1)^3}$.

15. Find the limit $\lim_{x\to 0} (1-3x)^{1/x}$.

16. Compute the derivative of the function $f(x) = (x+2)^{\sin x}$.