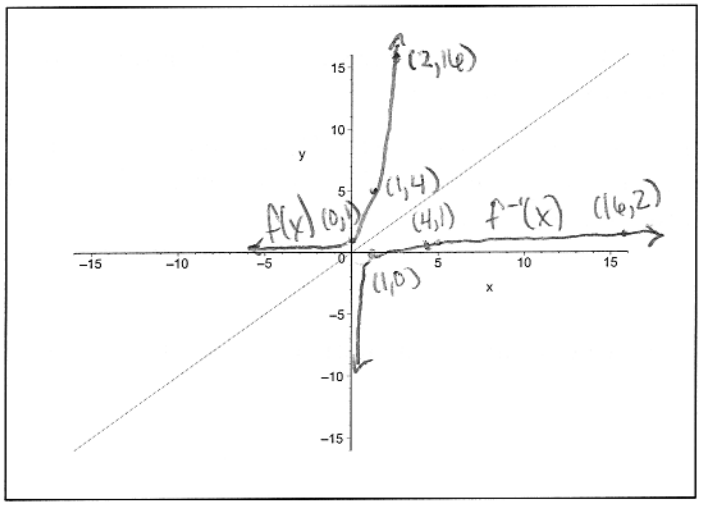
SHOW WORK FOR FULL CREDIT NO CALCULATORS

1. Let $f(x) = 4^x$.
   (a) What is the inverse of $f(x)$?
   (b) Plot and label $f(x)$ and its inverse on the axes below, and label at least three **pairs of points** (6 total), where a pair of points has one point on the graph of $y = f(x)$ and the corresponding point on the graph of the inverse.



(a) $f^{-1}(x) = \log_4 x$

2. Evaluate the integral $\displaystyle\int_0^{\pi/2} \cos x \cdot e^{\sin x}\, dx$.

$u = \sin x$ $\quad x = \frac{\pi}{2} \Rightarrow u = 1$

$\frac{du}{dx} = \cos x$ $\quad x = 0 \Rightarrow u = 0$

$du = \cos x\, dx$

$$\int_0^{\pi/2} \cos x\, e^{\sin x}\, dx = \int_0^1 e^u\, du = e^u\Big|_0^1 = e^1 - e^0 = e - 1$$

3. Differentiate the function $f(x) = x \cdot 5^{\sin(x)}$.

by quotient rule

$f'(x) = x\, 5^{\sin x} \ln 5 \cos x + 5^{\sin x}$

or $5^{\sin x}(\ln 5\ x \cos x + 1)$

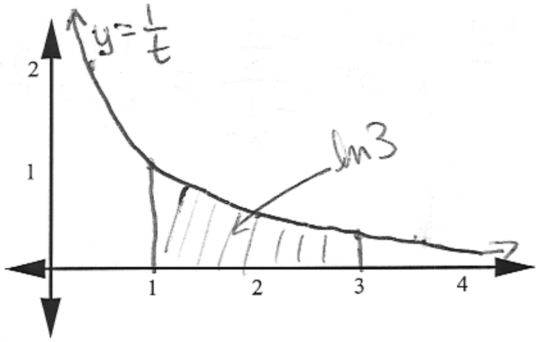
4. Simplify the expression $e^{\ln(\sin x) + \ln x - \ln y}$.

$= e^{\ln \sin x}\, e^{\ln x}\, e^{-\ln y}$

$= \sin x \cdot x \cdot e^{\ln y^{-1}}$

$= \frac{\sin x \cdot x}{y}$

5. Recall that by definition, $\ln x := \int_1^x \frac{1}{t}\, dt$. On the axes below, illustrate $\ln(3)$ as an area, and label **all relevant quantities**.



6. Using the Laws of Logarithms write the following quantity in the form $\ln(\cdot)$ (natural log of a single argument): $2\ln\cos(x) + 5\ln(x-4) - 10\ln(z+w)$

$= \ln\cos^2 x + \ln(x-4)^5 - \ln(z+w)^{10}$

$= \ln\left[\frac{\cos^2 x\,(x-4)^5}{(z+w)^{10}}\right]$

7. Evaluate the integral $\int \frac{\sec^2 x}{\tan x}\, dx$.

$u = \tan x$

$du = \sec^2 x\, dx$

$\int \frac{\sec^2 x}{\tan x}\, dx = \int \frac{du}{u} = \ln|u| + C = \ln|\tan x| + C$

8. Differentiate the function $f(x) = (\ln x)^3$.

$f'(x) = 3(\ln x)^2\left(\frac{1}{x}\right)$

or $\frac{3}{x}(\ln x)^2$

9. Find the limit $\lim_{x\to 0^+} x\cdot(\ln x)^2$.

$\lim_{x\to 0^+} x = 0$ $\quad \lim_{x\to 0^+} (\ln x)^2 = \infty$

IF type $0\cdot\infty$

$$\lim_{x\to 0^+} x(\ln x)^2 = \lim_{x\to 0^+} \frac{(\ln x)^2}{\frac{1}{x}}$$

$$\overset{L'H}{=} \lim_{x\to 0^+} \frac{2\ln x\left(\frac{1}{x}\right)}{-\frac{1}{x^2}}$$

$$= \lim_{x\to 0^+} \frac{-2\ln x}{\frac{1}{x}} \quad \text{IF type } \frac{\infty}{\infty}$$

$$\overset{L'H}{=} \lim_{x\to 0^+} \frac{-\frac{2}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} 2x = 0.$$

10. Find the limit $\lim_{x\to 0^+} \frac{2^{x^2+1}-1}{x^4}$.

$\lim_{x\to 0^+} 2^{x^2+1} - 1 = 2-1 = 1$

$\lim_{x\to 0^+} x^4 = 0$ $\quad$ <u>not</u> an indi. form

since $x\to 0$ from right,

$\lim_{x\to 0^+} \frac{1}{x^4} = \infty$,

and so $\lim_{x\to 0^+} \frac{2^{x^2+1}-1}{x^4} = \infty$.

11. Find the limit $\lim_{x\to 0} (1-x)^{2/x}$.

$\lim_{x\to 0} (1-x) = 1$ $\quad \lim_{x\to 0} \frac{2}{x} = \infty$ or $-\infty$

IF type 1^∞ or $1^{-\infty}$

$$\lim_{x\to 0} (1-x)^{2/x}$$

$$= \lim_{x\to 0} \exp\left(\ln(1-x)^{2/x}\right)$$

$$= \lim_{x\to 0} \exp\left(\frac{2}{x}\ln(1-x)\right)$$

$$= \exp\Big(\lim_{x\to 0} \underbrace{\frac{2\ln(1-x)}{x}}_{\text{IF } \frac{0}{0}}\Big)$$

$$\overset{L'H}{=} \exp\left(\lim_{x\to 0} \frac{\frac{-2}{1-x}}{1}\right)$$

$$= \exp(-2) \text{ or } e^{-2}.$$

12. Compute the derivative of the function $f(x) = (\cos x)^{x-5}$.

by logarithmic differentiation

$y = (\cos x)^{x-5}$

$\ln y = \ln(\cos x)^{x-5}$

$\ln y = (x-5)\ln\cos x$

$$\frac{y'}{y} = (x-5)\left(\frac{-\sin x}{\cos x}\right) + \ln\cos x$$

13. If $f(x) = x + e^x$ and $g(x)$ is the inverse of $f(x)$, find $g(1)$.

(from cover) $g(1) = x \Leftrightarrow f(x) = 1$

$f(0) = 0 + e^0 = 1$

so $g(1) = 0$.

14. Let $f(x) = \ln(3x - 9)$.
(a) What is the largest possible domain for $f(x)$? (Show calculation.)
(b) Prove that $f(x)$ is one-to-one on this domain by considering $f'(x)$. (Half-credit for a correct sketch plus the horizontal line test.)

(a) domain of $y = \ln(x)$ is $(0, \infty)$.

$3x - 9 > 0$ when

$3x > 9$

$x > 3$.

Therefore domain of $f(x)$ is $(3, \infty)$.

(b) $f'(x) = \dfrac{3}{3x-9}$

which is > 0 on $(3, \infty)$.

Therefore $f(x)$ is increasing on its domain and thus 1-1.

15. Find a formula for the inverse of the function $f(x) = \dfrac{2x-3}{3x+8}$.

$y = \dfrac{2x-3}{3x+8}$

$(3x+8)y = 2x - 3$

$3xy + 8y = 2x - 3$

$3xy - 2x = -8y - 3$

$x(3y-2) = -8y - 3$

$x = \dfrac{-8y-3}{3y-2}$

therefore $f^{-1}(x) = \dfrac{-8x-3}{3x-2}$

16. If $f(x) = x + \ln x$ and $g(x)$ is the inverse of $f(x)$, find the **derivative of** g **at** 1, $g'(1)$.

(from cover) $g'(1) = \dfrac{1}{f'(g(1))}$

$f'(x) = 1 + \dfrac{1}{x}$.

(from cover) $g(1) = x \Leftrightarrow f(x) = 1$.

by inspection, $f(1) = 1 + \ln 1 = 1$,

so $g(1) = 1$.

so $g'(1) = \dfrac{1}{f'(g(1))} = \dfrac{1}{f'(1)} = \dfrac{1}{1+\frac{1}{1}} = \dfrac{1}{2}$.