SHOW WORK FOR FULL CREDIT NO CALCULATORS Form 1a.

1. If $f(x) = x + \ln x$ and $g(x)$ is the inverse of $f(x)$, find $g(1)$.

(from cover) $g(1) = x \iff f(x) = 1$.
By inspection, $f(1) = 1 + \ln 1 = 1$
therefore $g(1) = 1$.

2. Let $f(x) = \sqrt{2x-4}$.
(a) What is the largest possible domain for $f(x)$? (Show calculation.)
(b) Prove that $f(x)$ is one-to-one on this domain by considering $f'(x)$. (Half-credit for a correct sketch plus the horizontal line test.)

(a) The domain of $y = \sqrt{x}$ is $[0, \infty)$
$2x - 4 \geq 0$ when
$2x \geq 4$
$x \geq 2$
Thus the domain of $f(x)$ is $[2, \infty)$.

(b) $f'(x) = \frac{1}{2}(2x-4)^{-\frac{1}{2}}(2)$
$= \dfrac{1}{\sqrt{2x-4}}$

$f'(x) > 0$ on all of $(2, \infty)$, and the endpoint at $x = 2$ doesn't affect the fact that $f(x)$ is increasing on its entire domain and is thus 1-1.

3. Find a formula for the inverse of the function $f(x) = \dfrac{3x-5}{4x+5}$.

$y = \dfrac{3x-5}{4x+5}$
$(4x+5)y = 3x-5$
$4xy + 5y = 3x - 5$
$4xy - 3x = -5y - 5$
$x(4y-3) = -5y - 5$

$x = \dfrac{-5y-5}{4y-3}$

by exchanging x & y,
$f^{-1}(x) = \dfrac{-5x-5}{4x-3}$

4. If $f(x) = x + e^x$ and $g(x)$ is the inverse of $f(x)$, find the **derivative of** g **at 1**, $g'(1)$.

(from cover) $g'(a) = \dfrac{1}{f'(g(a))}$

(from cover) $g(1) = x \iff f(x) = 1$
by inspection, $f(0) = 0 + e^0 = 1$
therefore $g(1) = 0$.

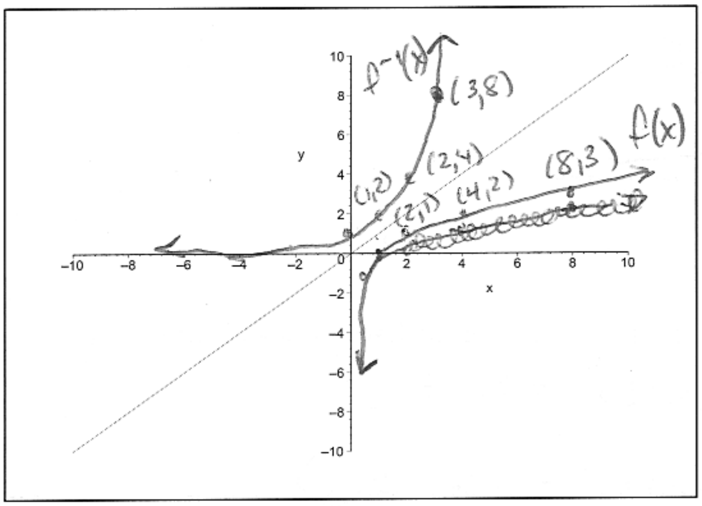
$f'(x) = 1 + e^x$
Therefore
$g'(1) = \dfrac{1}{f'(g(1))} = \dfrac{1}{f'(0)}$
$= \dfrac{1}{1+e^0} = \dfrac{1}{2}$

5. Let $f(x) = \log_2 x$.

(a) What is the inverse of $f(x)$?

(b) Plot and label $f(x)$ and its inverse on the axes below, and label at least three **pairs of points** (6 total), where a pair of points has one point on the graph of $y = f(x)$ and the corresponding point on the graph of the inverse.

(a) $f^{-1}(x) = 2^x$



6. Differentiate the function $f(x) = x \cdot e^{\sin x}$.

$f'(x) = xe^{\sin x}\cos x + e^{\sin x}$

7. Evaluate the integral $\int_0^1 x \cdot 3^{x^2}\, dx$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$x = 0 \Rightarrow u = 0$

$x = 1 \Rightarrow u = 1$

$\int_0^1 x\, 3^{x^2}\, dx$

$= \int_0^1 3^u \frac{du}{2}$

$= \frac{1}{2}\int_0^1 3^u\, du$

$= \frac{1}{2\ln 3} 3^u \Big|_0^1$

$= \frac{1}{2\ln 3}(3^1 - 3^0)$

$= \frac{1}{2\ln 3}(3-1) = \frac{1}{\ln 3}$

8. Simplify the expression $\ln\left(e^{\sin x - x} e^y\right)$.

$\ln\left(e^{\sin x - x} e^y\right) = \ln e^{\sin x - x} + \ln e^y$

$= \ln e^{\sin x} e^{-x} + \ln e^y$

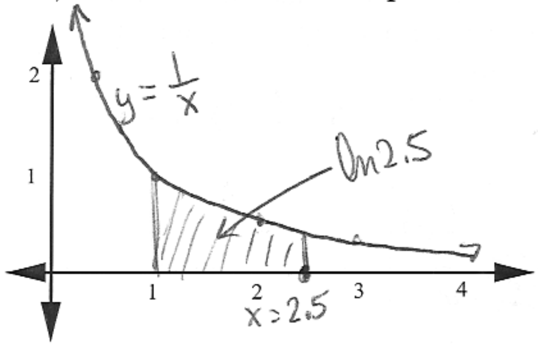
$= \ln e^{\sin x} + \ln e^{-x} + \ln e^y$

$= \sin x - x + y$

(or $= \ln e^{\sin x - x + y}$

$= \sin x - x + y$)

9. Recall that by definition, $\ln x := \int_1^x \frac{1}{t}\,dt$. On the axes below, illustrate $\ln(2.5)$ as an area, and label **all relevant quantities**.



10. Completely expand the following expression using the Laws of Logarithms: $\ln\left(\frac{(\sin x)^2}{(x+2)^4(z-y)^5}\right)$

$$= \ln(\sin x)^2 - \ln(x+2)^4 + \ln(z-y)^5$$

$$= 2\ln(\sin x) - 4\ln(x+2) + 5\ln(z-y)$$

11. Differentiate the function $f(x) = \ln(\tan x)$.

$$f'(x) = \frac{\sec^2 x}{\tan x}$$

12. Evaluate the integral $\int \frac{(\ln x)^3}{x}\,dx$.

$$u = \ln x$$

$$\frac{du}{dx} = \cancel{3(\ln x)^2}\ \frac{1}{x}$$

$$du = \frac{dx}{x}$$

$$\int \frac{(\ln x)^3}{x}dx = \int u^3\,du = \frac{u^4}{4} + C = \frac{(\ln x)^4}{4} + C$$

13. Find the limit $\lim_{x\to\infty} \frac{e^{x^2}}{x^4}$.

$\lim_{x\to\infty} e^{x^2} = \infty$

$\lim_{x\to\infty} x^4 = \infty$

IF $\frac{\infty}{\infty}$

$$\lim_{x\to\infty} \frac{e^{x^2}}{x^4} \overset{L'H}{=} \lim_{x\to\infty} \frac{2xe^{x^2}}{4x^3} = \lim_{x\to\infty} \frac{e^{x^2}}{2x^2} \quad \text{IF } \frac{\infty}{\infty}$$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{2xe^{x^2}}{4x} = \lim_{x\to\infty} \frac{e^{x^2}}{2} = \infty$$

14. Find the limit $\lim_{x\to 1^+} \frac{\log_2(x^2+1)}{(x-1)^3}$.

$\lim_{x\to 1^+} \log_2(x^2+1) = \log_2(2) = 1$

$\lim_{x\to 1^+} (x-1)^3 = 0$

not an Indeterminate form.

since $x\to 1$ from the right

$\lim_{x\to 1^+} \frac{1}{(x-1)^3} = \infty$

and $\lim_{x\to 1^+} \frac{\log_2(x^2+1)}{(x-1)^3} = \infty$

15. Find the limit $\lim_{x\to 0} (1-3x)^{1/x}$.

$\lim_{x\to 0} (1-3x) = 1$

$\lim_{x\to 0} \frac{1}{x} = \infty$ or $-\infty$

IF type 1^∞ or $1^{-\infty}$

$$\lim_{x\to 0} (1-3x)^{1/x} = \lim_{x\to 0} \exp\left(\ln (1-3x)^{1/x}\right)$$

$$= \lim_{x\to 0} \exp\left(\tfrac{1}{x} \ln(1-3x)\right)$$

$$= \exp\Big(\underbrace{\lim_{x\to 0} \frac{\ln(1-3x)}{x}}_{\text{IF type } \frac{0}{0}}\Big)$$

$$\overset{L'H}{=} \exp\left(\lim_{x\to 0} \frac{\frac{-3}{1-3x}}{1}\right)$$

$$= \exp(-3) = e^{-3}.$$

16. Compute the derivative of the function $f(x) = (x+2)^{\sin x}$.

by logarithmic diffentiation.

$y = (x+2)^{\sin x}$

$\ln y = \ln(x+2)^{\sin x}$

$\ln y = \sin x \ln(x+2)$

$\frac{y'}{y} = \sin x \frac{1}{x+2} + \cos x \ln(x+2)$

$y' = y\left[\frac{\sin x}{x+2} + \cos x \ln(x+2)\right]$

$f'(x) = (x+2)^{\sin x}\left[\frac{\sin x}{x+2} + \cos x \ln(x+2)\right]$