Integral Test Estimation Formula

Here is a summary of the formula we derived on 11/5/03 for estimating a series derived from a continuous, nonnegative, decreasing function.

Suppose the following:

- $f(x)$ is a continuous, nonnegative, decreasing function,
- the series $\sum_{n=1}^{\infty} a_n$ is defined by $a_n = f(n)$ for all n,
- we define $s_n = \sum_{i=1}^{n} a_i$ to be the nth partial sum, and
- both $\sum_{n=1}^{\infty} a_n$ and $\int_0^{\infty} f(x)\,dx$ converge, so that $\sum_{n=1}^{\infty} a_n = s$.

Then the quantity

$$s_n + \frac{\int_n^{\infty} f(x)\,dx + \int_{n+1}^{\infty} f(x)\,dx}{2}$$

is guaranteed to be within $\frac{a_n}{2}$ of s.

Note: In our derivation, we actually guaranteed that it would be within

$$\frac{\int_n^{\infty} f(x)\,dx - \int_{n+1}^{\infty} f(x)\,dx}{2} = \frac{\int_n^{n+1} f(x)\,dx}{2}$$

of s, but because $f(x)$ is decreasing, we know that $\int_n^{n+1} f(x)\,dx \leq a_n$.